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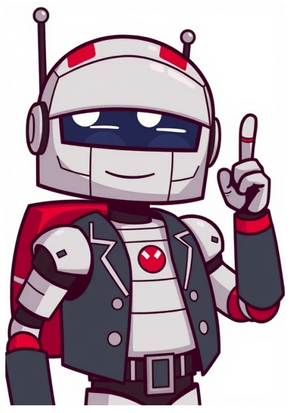

Forget everything you know about numbers. In fact, forget you even know what a number is. This is where mathematics starts. Instead of math with numbers, we will now think about math with "things". What is a set? Well, simply put, it's a collection. First we specify a common property among "things" (we define this word later) and then we gather up all the "things" that have this common property. For example, the items you wear: hat, shirt, jacket, pants, and so on. I'm sure you could come up with at least a hundred. This is known as a set. Or another example is types of fingers. This set includes index, middle, ring, and pinky. So it is just things grouped together with a certain property in common. Notation There is a fairly simple notation for sets. We simply list each element (or "member") separated by a comma, and then put some curly brackets around the whole thing: The curly brackets { } are sometimes called "set brackets" or "braces". This is the notation for the two previous examples: {socks, shoes, watches, shirts, ...}{index, middle, ring, pinky} Notice how the first example has the "..." (three dots together). The three dots ... are called an ellipsis, and mean "continue on". So that means the first example continues on ... for infinity. (OK, there isn't really an infinite amount of things you could wear, but I'm not entirely sure about that! After an hour of thinking of different things, I'm still not sure. So let's just say it is infinite for this example.) So: The first set {socks, shoes, watches, shirts, ...} we call an infinite set, the second set {index, middle, ring, pinky} we call a finite set. But sometimes the "..." can be used in the middle to save writing long lists: Example: the set of letters: {a, b, c, ..., x, y, z} In this case it is a finite set (there are only 26 letters, right?) Numerical Sets So what does this have to do with mathematics? When we define a set, all we have to specify is a common characteristic. Who says we can't do so with numbers? Set of even numbers: {..., 4, 2, 0, 2, 4, ...}Set of odd numbers: {..., 3, 1, 1, 3, ...}Set of prime numbers: {2, 3, 5, 7, 11, 13, 17, ...}Positive multiples of 3 that are less than 10: {3, 6, 9}And so on. We can come up with all different types of sets. We can also define a set by its properties, such as {x|x>0} which means "the set of all x's, such that x is greater than 0", see Set-Builder Notation to learn more. And we can have sets of numbers that have no common property, they are just defined that way. For example: {2, 3, 6, 828, 3839, 8827}{4, 5, 6, 10, 21}{2, 949, 48282, 42882959, 119484203}Are all sets that I just randomly banged on my keyboard to produce. Why are Sets Important? Sets are the fundamental property of mathematics. Now as a word of warning, sets, by themselves, seem pretty pointless. But it's only when we apply sets in different situations do they become the powerful building block of mathematics that they are. Math can get amazingly complicated quite fast. Graph Theory, Abstract Algebra, Real Analysis, Complex Analysis, Linear Algebra, Number Theory, and the list goes on. But there is one thing that all of these share in common: Sets. Universal Set At the start we used the word "things" in quotes. We call this the universal set. It's a set that contains everything. Well, not exactly everything. Everything that is relevant to our question. In Number Theory the universal set is all the integers, as Number Theory is simply the study of integers. But in Calculus (also known as real analysis), the universal set is almost always the real numbers. And in complex analysis, you guessed it, the universal set is the complex numbers. Some More Notation When talking about sets, it is fairly standard to use Capital Letters to represent the set, and lowercase letters to represent an element in that set.So for example, A is a set, and a is an element in A. Same with B and b, and C and c. Now you don't have to listen to the standard, you can use something like m to represent a set without breaking any mathematical laws (watch out, you can get years in math jail for dividing by 0), but this notation is pretty nice and easy to follow, so why not? Also, when we say an element a is in a set A, we use the symbol to show it.And if something is not in a set use . Example: Set A is {1,2,3}. We can see that 1 A, but 5 A Equality Two sets are equal if they have precisely the same members. Now, at first glance they may not seem equal, so we may have to examine them closely! Example: Are A and B equal where: A is the set whose members are the first four positive whole numbers B = {4, 2, 1, 3} Let's check. They both contain 1. They both contain 2. And 3, And 4. And we have checked every element of both sets, so: Yes, they are equal! And the equals sign (=) is used to show equality, so we write: A = B A is {1, 2, 3} B is {3, 1, 2} Yes, they are equal! They both contain exactly the members 1, 2 and 3. It doesn't matter where each member appears, so long as it is there. Subsets When we define a set, if we take pieces of that set, we can form what is called a subset. A subset of this is {1, 2, 3}. Another subset is {3, 4} or even another is {1}, etc. But {1, 6} is not a subset, since it has an element (6) which is not in the parent set. In general: A is a subset of B if and only if every element of A is in B. So let's use this definition in some examples. 1 is in A, and 1 is in B as well. So far so good. 3 is in A and 3 is also in B. 4 is in A, and 4 is in B. That's all the elements of A, and every single one is in B, so we're done. Yes, A is a subset of B Note that 2 is in B, but 2 is not in A. But remember, that doesn't matter, we only look at the elements in A. Let's try a harder example. Well, we can't check every element in these sets, because they have an infinite number of elements. So we need to get an idea of what the elements look like in each, and then compare them. The sets are: A = {..., 8, 4, 0, 4, 8, ...} B = {..., 8, 6, 4, 2, 0, 2, 4, 6, 8, ...} By pairing off members of the two sets, we can see that every member of A is also a member of B, but not every member of B is a member of A: So: A is a subset of B, but B is not a subset of A Proper Subsets If we look at the defintion of subsets and let our mind wander a bit, we come to a weird conclusion. Let A be a set. Is every element of A in A? Well, umm, yes of course, right? So that means that A is a subset of A. It is a subset of itself! This doesn't seem very proper, does it? If we want our subsets to be proper we introduce (what else but) proper subsets: A is a proper subset of B if and only if every element of A is also in B, and there exists at least one element in B that is not in A. This little piece at the end is there to make sure that A is not a proper subset of itself: we say that B must have at least one extra element. {1, 2, 3} is a subset of {1, 2, 3}, but is not a proper subset of {1, 2, 3}. {1, 2, 3} is a proper subset of {1, 2, 3, 4} because the element 4 is not in the first set. Notice that when A is a proper subset of B then it is also a subset of B. Even More Notation When we say that A is a subset of B, we write A B Or we can say that A is not a subset of B by A B When we talk about proper subsets, we take out the line underneath and so it becomes A B or if we want to say the opposite A B Empty (or Null) Set This is probably the weirdest thing about sets. As an example, think of the set of piano keys on a guitar. "But wait!" you say, "There are no piano keys on a guitar!" And right you are. It is a set with no elements. This is known as the Empty Set (or Null Set).There aren't any elements in it. Not one. Zero. It is represented by Or by {} (a set with no elements) Some other examples of the empty set are the set of countries south of the south pole. So what's so weird about the empty set? Well, that part comes next. Empty Set and Subsets So let's go back to our definition of subsets. We have a set A. We won't define it any more than that, it could be any set. Is the empty set a subset of A? Going back to our definition of subsets, if every element in the empty set is also in A, then the empty set is a subset of A. But what if we have no elements? It takes an introduction to logic to understand this, but this statement is one that is "vacuously" or "trivially" true. A good way to think about it is: we can't find any elements in the empty set that aren't in A, so it must be that all elements in the empty set are in A. So the answer to the posed question is a resounding yes. The empty set is a subset of every set, including the empty set itself. Order No, not the order of the elements. In sets it does not matter what order the elements are in. Example: {1,2,3,4} is the same set as {3,1,4,2} When we say order in sets we mean the size of the set. Another (better) name for this is cardinality. A finite set has finite order (or cardinality). An infinite set has infinite order (or cardinality). For finite sets the order (or cardinality) is the number of elements. Example: {10, 20, 30, 40} has an order of 4. For infinite sets, all we can say is that the order is infinite. Oddly enough, we can say with sets that some infinities are larger than others, but this is a more advanced topic in sets. Arg! Not more notation! Nah, just kidding. No more notation. by Ricky Shadrachand Rod Pierce 366, 367, 368, 1051, 1052, 1053, 9070, 2424, 2425, 2426 Copyright 2024 Rod Pierce A set is a collection of well-defined objects that share some common property. It can be a group of any items, such as the names of the months in a year, the days in a week, or a list of variables or constants. Sets are named and represented in capital letters. Here are some examples of sets: A = {-5, -3, -1, 1, 3, 5} B = {2, 3, 5, 7, 11, 13, } Here are some standard sets in mathematics: Set of natural numbers; = {1, 2, 3, } Set of whole numbers; = {0, 1, 2, 3, } Set of integers; = {, -3, -2, -1, 0, 1, 2, 3, } Set of rational numbers; = {$\dfrac{x}{y}$ | x and y are integers and y 0} Set of irrational numbers; = {x is not rational} Set of real numbers; = In a set, we separate each term by a comma and then put curly brackets { } around the whole group. These curly brackets { } are called set brackets or braces. The terms or items present in the set are called elements or members of that set. The elements are represented by lowercase letters. The first set has 6 terms, but the second set has an uncountable number of terms. The three dots together are called ellipses, which means continue on. Sometimes, the ellipsis is used in the middle of terms to avoid writing many terms. For example, in the set C = {a, b, c, , z}, there are 26 letters, and thus there are 26 terms in set C. If A is a set containing elements a and b, A = {a, b}. Here, we also use the symbol (read as belongs to) to represent the element a in the set A. Since a is in the set A, but another element c is not in the set A, we can write a A and c A, read as a belongs to A and c does not belong to A. The number of elements present in a set is called the cardinal number, cardinality, or order of a set. For example, the cardinality of the set A of natural numbers A = {1, 2, 3, 4} is 4, which is written as n(A) = |A| = 4 There are some different set notations used to represent sets. Semantic notation describes a statement to represent the elements of a set. For example, a set A = {0, 1, 2, 3, 4, 5, 6, 7, 8} in semantic form is written as {a set of whole numbers less than 8} This is the most common way of representing sets. In roster notation, the elements are enclosed within curly brackets and are separated by commas. For example, the set of whole numbers less than 8 is represented in roster form as B = {0, 1, 2, 3, 4, 5, 6, 7, 8} In set-builder notation, a rule or statement describes the common property of all the elements. For example, a set of whole numbers less than 8 is represented as B = {x | x is a whole number and x < 8} A set that does not contain any element is called an empty set or null set. It is represented using the symbol , read as phi. For example, Set A = { } A set that contains only one element is called a singleton or unit set. For example, Set A = {0} and Set B = {9} A set that contains a finite or countable number of elements is called a finite set. For example, Set B = {1, 4, 9, 16, 25, 36, 49, 64, 81, 100} A set containing an infinite number of elements is called an infinite set. For example, the set of negative integers is {, -4, -3, -2, -1} If two sets contain the same number of elements, even if the elements are different, they are called equivalent sets. For example, if A = {-8, -4, -2, 0} and B = {p, q, r, s}, since n(A) = n(B), sets A and B are equivalent. If two sets contain the same elements, they are called equal sets. For example, if A = {4, 5, 6}, B = {4, 5, 6}, and C = {5, 6, 4}, then sets A, B, and C are equal (A = B = C) irrespective of the order of the elements. Two sets are said to be unequal if they have at least one unique element. For example, if A = {4, 5, 6} and B = {2, 3, 4}, then set A and B are unequal, which is written as A B. If A and B are two sets and at least one element from A is present in B or vice versa, then the two sets A and B are said to be overlapping. For example, if A = {1, 2, 3, 4} and B = {3, 4, 5, 6, 7}, then elements 3 and 4 are common in both sets A and B. Thus, A and B are overlapping sets. If A and B are two sets with no common elements in between, then sets A and B are called disjoint sets. For example, if A = {m, n, s, t} and B = {5, 6, 7, 8, 9}, then set A and set B have no common elements in between. They are disjoint sets. If A and B are two sets such that every element in A is also present in B, then set A is the subset of set B, represented by A B. Here, B is known as the superset of set A, which is represented as B A. For example, if A = {2, 3, 5, 7, 11} and B = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}, since all the elements of A are present in B, A is a subset of set B, written as A B. Also, set B is the superset of set A (B A.) Again, if X = {1, 2, 3, 4}, Y = {1, 2, 3, 4}, and Z = {1, 2, 3, 4, 5}, then every element of X is present in Y, whereas element 5 of the set Z is not in set X. Thus, set X is a proper subset of set Z. In general, A is a proper subset of B if and only if every element of A is also in B, but there is at least one element in B that is not present in A. It is represented as A B. A power set is a set of all subsets that the set could contain. For example, if A = {0, 1}, the power set of A is represented by P(A). Here, P(A) = {, {0}, {1}, {0, 1}} A universal set, represented by the letter U, is the collection of all the elements relevant to a certain condition. For example, if U is the set of real numbers, the sets of natural numbers and rational numbers are the subsets of this universal set. Here is the list of different types of sets we learned. Here is the list of all the symbols used to define the elements in a set. The union of sets, represented by A B, lists all the elements in sets A and B or the elements present in both sets A and B. For example, if A = {4, 5} and B = {5, 6, 7}, then A B = {4, 5} {5, 6, 7} = {4, 5, 6, 7} The intersection of sets, represented by A B, lists the common elements of sets A and B. For example, if A = {4, 5} and B = {5, 6, 7}, then A B = {4, 5} {5, 6, 7} = {5} The set difference represents the elements in set A that are not present in set B. It is represented by A B. For example, A = {1, 2, 3, 4} and B = {3, 4, 5, 6}. A B = {1, 2} The complement of a set A, denoted by A, represents all elements in the universal set that are absent in set A. Also, A is represented as U A, the difference between the elements of the universal set and set A. For example, if the universal set is U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and set A = {2, 3, 4, 5, 6}, then A = {1, 7, 8, 9, 10} The cartesian product of two sets, A and B, written as A B, is the product of two non-empty sets. Here, the ordered pairs of elements are obtained, which means the first element is obtained from the first set and the second element from the second set. For example, {5, 6} {7, 8} = {(5, 7), (6, 7), (5, 8), (6, 8)} The symmetric difference between two sets, A and B, represented by A B, represents the remaining elements after removing the common elements from the union of the two sets. Mathematically, it is written as A B = (A B) (A B) A B = (A B) (B A), which represents the elements of only set A and only set B. Here is a summary of the commonly used set operations: If A and B are two overlapping sets, then n(A B)n(A) + n(B) n(A B)n(A B)n(A) + n(B) n(A B)n(A)n(A B) + n(A B) n(B)n(B)n(A B) + n(A B) n(A)n(A B)n(A B) n(B)n(A B)n(A) n(A B)n(A B C)n(A) + n(B) + n(C) n(A B) n(B C) n(C A) + n(A B C) If A and B are two disjoint sets, then n(A B)n(A) + n(B)(A B)n(A B)n(A) If A, B, and C are three sets, they follow the following properties. Commutative PropertyA B = B AA B = B AAssociative property(A B) C = A (B C)(A B) C = A (B C)Distributive PropertyA (B C) = (A B) (A C)A (B C) = (A B) (A C)Identity PropertyA = AA = AComplement PropertyA A = Idempotent PropertyA A = AA A = A Sets are often represented pictorially using a Venn diagram for ease of understanding. It also illustrates how the given sets relate to each other. In a Venn diagram, the elements of each set are enclosed in a circle. Sometimes, a rectangle surrounds the circles, representing the universal set. Here are the commonly used set operations with their Venn diagrams: The last two operations are known as De Morgans Laws. For more, follow De Morgans Law. If A and B are two sets such that n(A) = 5, n(B) = 9, and n(A B) = 4, then determine the value of n(A B).Solution:As we know, n(A B) = n(A) + n(B) n(A B)Here, n(A) = 5, n(B) = 9, and n(A B) = 4Thus, n(A B) = 5 + 9 4 = 10 If Set A = {10, 20, 30, 40}, Set B = {5, 10, 15, 20, 25, 30, 35, 40}, and the universal set is U = , then find the following using sets formulas:a) A Bb) A Bc) Is A B?Solution:Here,A = {10, 20, 30, 40} and B = {5, 10, 15, 20, 25, 30, 35, 40}Thus,a) A B = {5, 10, 15, 20, 25, 30, 35, 40}b) A B = {10, 20, 30, 40}c) SInce all the elements of set A are present in set B, thus A B.Yes, A B. Express the given set in the set-builder notation: A = {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}Solution:GivenA = {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}Thus, by representing the given set A in the set-builder form, we getA = {x2 | x and x 10} Last modified on July 19th, 2024 We commonly use the terms like a complete set of novels or a set of cutlery in day-to-day life. What do we mean by the term set here? It simply defines a collection of objects or things of the same type. Sets in math are also defined in the similar context. In mathematics, a set is defined as a collection of distinct, well-defined objects forming a group. There can be any number of items, be it a collection of whole numbers, months of a year, types of birds, and so on. Each item in the set is known as an element of the set. We use curly brackets while writing a set. Consider an example of a set. $\text{A} = \left\{1, 3, 5, 7, 9\right\}$. It has five elements. It is a set of odd numbers less from 1 to 10. What do we mean by well-defined objects? Consider an example. A collection of odd natural numbers less than 20 is defined, but a collection of brave students in a class is not defined. We can represent a collection of odd natural numbers less than 20 in the form of a set as $\text{B} = \left\{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\right\}$. The number of elements in the set are denoted by n(A) where A is a set. Example: $\text{A} = \left\{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\right\}$. $n(A) = 10$. The other word used for the number of elements in the set is called its cardinality. More Worksheets Elements or members are the terms or items present in a set. They are enclosed in curly brackets and separated by commas. To represent that an element is contained in a set, we use the symbol $\in$. It is read as belongs to. Suppose we have a set of even natural numbers less than 10. $\text{A} = \left\{2, 4, 6, 8\right\}$. Here, $2 \in A$ but $3 otin A$. We represent the sets in different ways. The only difference is in the way in which the elements are listed. The different forms of representing sets are discussed below. The most familiar and easy form used to represent sets is the roster form, in which the elements are enclosed in curly brackets and are separated by commas such as $\text{B} = \left\{1, 4, 9, 16, 25\right\}$, which is the collection of the square of consecutive numbers less than 30. The order of the elements does not matter and there can be an infinite number of elements in a set, which we define using a series of dots at the end of the last element. There are two types of sets in Roster Form: Finite Roster Notation of Sets in which there are elements that can be counted, such as $\text{A} = \left\{5, 10, 15, 20, 25\right\}$ (The multiples of 5 less than 30.) Infinite Roster Notation of Sets in which the elements can not be counted, such as $\text{B} = \left\{4, 8, 12, 16 \right\}$ (The multiples of 4) The set builder notation has a particular rule that describes the common feature of all the elements of a set. It uses a vertical bar in its representation along with a text describing the character of the elements, such as $\text{A} = \left\{ \text{x}\; |\; \text{x} \; \text{is a prime number}, x \le 20\right\}$. According to the statement, all the elements of the set are prime numbers less than or equal to 20. We use : place of the | sometimes. Lets discuss different types of sets. When a set has only one element, it is known as a singleton set. Set $\text{A} = \left\{ \text{x}\; |\; \text{x}\; \text{is a whole number between}\;12\; \text{and}\;14\right\} = \left\{13\right\}$. When a set does not contain any element, it is known as a null or an empty set. It is denoted by the symbol Φ and it is read as phi. Set $\text{B} = =$ Integers between 1 and $2 = \Phi$ Two sets are said to be equal if they have the same elements in them. Suppose there are two sets $\text{A} = \left\{1, 4, 5\right\}$ and $\text{B} = \left\{1, 4, 5\right\}$. Here, $\text{A} = \text{B}$ because each element is the same. Two sets are said to be unequal if they have at least one different element. Suppose we have two sets $\text{A} = \left\{1, 2, 5\right\}$ and $\text{B} = \left\{1, 2, 4\right\}$. Here, $\text{A} eq \text{B}$ as $5 \in \text{A}$ but $5 otin \text{B}$. Equivalent sets are the two sets that have the same number of elements, even if the elements are different. Suppose we have two sets $\text{A} = \left\{10, 11, 12, 13\right\}$ and $\text{B} = \left\{January, February, March, April\right\}$. Since $n(A) = n(B)$, A and B are equivalent sets. If at least one element from set A is present in set B, then the sets are said to be overlapping. Example: $\text{A} = \left\{2, 3, 6\right\} \text{B} = \left\{6, 8, 12\right\}$. 6 is present in set A and set B. So, A and B are overlapping sets. If every element in set A is also present in set B, then set A is known as the subset of set B, which is denoted by $\text{A} \subseteq \text{B}$ and B is known as the superset of set A, which is denoted by $\text{B} \supeteq \text{A}$. Example: $\text{A} = \left\{1, 4, 7, 10, 12\right\}\; \text{B} = \left\{1, 4, 6, 7, 8, 10, 11, 12, 13\right\}$ $\text{A} \subseteq \text{B}$, since all the elements of A are present in B. Also, set B is the superset of set A. The collection of all the elements in regard to a particular subject is known as a universal set which is denoted by the letter U. Suppose we have a set U as the set of all the natural numbers. So, the set of even numbers, set of odd numbers, set of prime numbers is a subset of the universal set. Two sets are known as disjoint sets if they have no common elements in both sets. Consider two sets $\text{A} = \left\{5, 6, 7\right\}$ and $\text{B} = \left\{2, 3, 4\right\}$. Here, set A and set B are disjoint sets as they have no elements in common. The set of all subsets that a set could contain is known as the power set. Suppose we have a set $\text{A} = \left\{2, 3\right\}$. Power set of A is $= \left\{\left\{\varnothing\right\}, \left\{2\right\}, \left\{3\right\}, \left\{2,3\right\}\right\}$. If n is the number of elements in a set, then the number of subsets $=2^{n}$. The pictorial representation of sets represented as circles is known as the Venn diagram. The elements of the sets are inside the circles. The rectangle that encloses the circles represents the universal set. The Venn diagram represents how the sets are related to each other. There are some operations on sets given below: B. Suppose we have two sets $\text{A} = \left\{1, 2, 3\right\}$ and $\text{B} = \left\{3, 4, 5\right\}$ $\text{A}\; \text{U}\; \text{B} = \left\{1, 2, 3, 4, 5\right\}$ The intersection of sets is denoted by $\text{A} \cap \text{B}$ has the elements which are common in both set A and set B. Suppose we have two sets $\text{A} = \left\{1, 3\right\}$ and $\text{B} = \left\{3, 4\right\}$ $\text{A} \cap \text{B} = {3}$ We denote the set difference by $\text{A} \text{B}$, which has the elements in set A that are not present in set B. Suppose we have two sets, i.e., $\text{A} = \left\{3, 4, 5\right\}$ and $\text{B} = \left\{5, 6, 7\right\}$ $\text{A} \;\; \text{B} = \left\{3, 4\right\}$ The complement of a set A is denoted by A, which is the set of all elements in the universal set that are not present in set A. A can also be represented as $\text{U} \;\; \text{A}$, i.e., the difference in the elements of the universal set and set A. Suppose $\text{U} =$ Set of Natural Numbers and $\text{A} =$ Set of Prime Numbers So, $\text{U} \;\; \text{A} =$ Set of all Non-prime Numbers. There are some set formulas that we can use to find the number of elements. For sets A and B, $n(A\; U\; B) = n(A) + n(B) n(A \cap B)$ $n(A B) = n(A U B) n(B)$ $n(A B) = n(A) n(A \cap B)$ Here are the properties of sets: Set PropertyRepresentationCommutative PropertyA B = B AA B = B AAssociative PropertyA (B C) = (A B) CA (B C) = (A B) CDistributive PropertyA (B C) = (A B) (A C)A (B C) = (A B) (A C)Identity PropertyA = AA U = AComplement PropertyA A = UIdempotent PropertyA A = AA A = AProperties of and UA = AA = A U = UA U = A 1. How many elements are there in the set $\text{A} = \left\{ \text{x}\; : \text{x}\; \text{is a perfect square less than 30}\right\}$? Solution: $\text{A} = \left\{1, 4, 9, 16, 25\right\}$ $n(A) = 5$ 2. Arrange the set $A = \left\{ y : y^{2} = 36 ; y\; \text{is an integer}\right\}$ in roster form. Solution: $y^{2} = 36 \Rightarrow y^{2} 36 = 0 \Rightarrow y = \pm 6$ $\text{A} = { \; 6, 6}$ 3. Write the set $\text{B} = \left\{1, 2, 5, 10, 17\right\}$ in set builder form. Solution: $0^{2} + 1 = 1$ $1^{2} + 1 = 2$ $2^{2} + 1 = 5$ $3^{2} + 1 = 10$ $4^{2} + 1 = 17$ So, in roaster form $\text{B} = \left\{y : y^{2} + 1, y \lt 5\right\}$ 4. If A is a set of prime numbers and B is a set of even numbers. What will be $\text{A} \cap \text{B}$? Solution: Only 2 is the number, which is a prime number as well as an even number. So, $\text{A} \cap \text{B} = {2}$. 5. Draw a Venn diagram for the sets $\text{A} = \left\{1, 3, 6, 9, 11, 15, 17\right\}$ and $\text{B} = \left\{2, 4, 9, 11, 13, 17\right\}$. Solution: Attend this quiz & Test your knowledge.Correct answer is: Null SetThere is no element in $\left\{\right\}$. So, it is a null set.$\left\{ 0, 1, 2, 3, 5, 6, 7\right\}$$\left\{ 0, 2, 3, 5\right\}$$\left\{\; \; 1, 1, 7\right\}$$\left\{\; \; 1, 0, 1, 2, 3, 5, 6, 7\right\}$Correct answer is: $\left\{\; \; 1, 0, 1, 2, 3, 5, 6, 7\right\}$$\text{A}\; \text{U}\; \text{B} = \left\{\; \; 1, 0, 1, 2, 3, 5, 6, 7\right\}$$\text{P} \;\; \text{Q}$Correct answer is: $\text{P} \;\; \text{Q}$The elements in $\left\{ \;\; 10, \;\; 3, \;\; 1\right\}$ are elements that are in Set P but not in Set Q.Correct answer is: 8Number of subsets $= 2^{n} = 2^{3} = 8$Correct answer is: 27$n(A \cup B) = n(A) + n(B) \;-\; n(AB) = 16 + 18 \;-\; 7 = 27$ What are sets formula if sets are disjoint? For disjoint sets A and B; $n(A U B) = n(A) + n(B)$ $A \cap B = \varnothing$ $n(A B) = n(A)$ No. There is one element inside the brackets. So, it is a singleton set, not a null set. What is a Cartesian Product in sets? Cartesian Product of Set A and B is defined as an ordered pair (x, y) where $x \in A$ and $y \in B$. How is set theory applicable in daily lives? In real life, sets are used in bookshelves while arranging the books according to alphabetic order or genre; closets where dresses, tops, jeans are kept as a set, etc. Does the union of two sets include the intersection of the sets? Yes, the union of two sets includes the intersection of the sets. Union of set A and B includes the elements either in set A or in set B or in both set A and B. home / probability and statistics / set theory / setA set is a collection of mathematical objects. Mathematical objects can range from points in space to shapes, numbers, symbols, variables, other sets, and more. Each object in a set is referred to as an element. Below are a few examples of different types of sets. A = {a, b, c, d, e} is a set made up of lower-case letters. B = {n|n N, 1 n 10} is a set made up of the first 10 natural numbers. C = {green, blue, red, yellow, white, black, purple} is a set made up of colors. M = {Jan, Feb, Mar, Apr, May, Jun, Jul, Aug, Sep, Oct, Nov, Dec} is a set made up of the 12 months in a year. P = the set of polygons shown in the figure below: = the universal set that contains subsets A and B, shown in the venn diagram below. Ways to define a set As can be seen in the list above, there are a number of different ways to define a set. Definition through rules A set can be defined using rules to determine the elements of the set. The rules must be strict such that the set is well-defined. For example, "the set of all even integers," is a well-defined set. There is nothing ambiguous about the set because the even integers are well-defined. In contrast to the above, "the set of all young people who like ice cream," is not a well-defined set. "Young people" is an ambiguous term that doesn't specify age while the degrees to which people "like" ice cream cannot be quantified. Roster notation Roster notation defines a set using curly brackets to contain a list of the elements that make up the set, separated by commas. The set of all even integers can be defined in roster notation as: A = {..., -6, -4, -2, 0, 2, 4, 6, ...} Set-builder notation Set-builder notation is similar to roster notation in its use of brackets, but rather than listing elements, conditions expressed using specific symbols (described in the table below) are applied to a larger set in order to specify a smaller set. For example, the set of all even integers is a subset of the set of all integers, and can be expressed in set-builder notation as A = {2a|a} where a is an integer, and "|" is read as "such that." Set-builder notation can also be expressed in other ways. For example, the set of all integers greater than 12 could be expressed as: B = {b|b>12} Symbols used in set theory There are many different symbols that are used within set theory. The table below includes some of the most common symbols. Symbol Definition/meaning Example { } Indicates a collection of elements {1, 3, 7, 9} Empty set - set contains no elements/td> {} ... Indicates that the set continues the pattern in the corresponding direction (towards negative (left) or positive (right) infinity) {..., -9, -7, -3, -1, 1, 3, 7, 9, ...} | "Such that" a|a > 3 - "a is an integer such that a is greater than 3" "and" or "intersection" A = {1, 2, 3, 4}B = {4, 5, 6}AB = {4} "or" or "union" A = {1, 2, 3, 4}B = {4, 5, 6}AB = {1, 2, 3, 4, 5, 6} Subset - A is a subset of B if all its elements are included in B {1, 2} {1, 2, 3, 4, 5}{1, 2, 3, 4, 5} {1, 2, 3, 4, 5} Proper/strict subset - A is a proper subset of B if A is a subset of B, but not equal to B {1} {1, 2, 3, 4, 5}{1, 2} {1, 2, 3, 4, 5}{1, 2, 3} {1, 2, 3, 4, 5}{1, 2, 3, 4} {1, 2, 3, 4, 5} Element of - indicates that the object on the left of the symbol is an element of the object on the right x - "x is an element of the rational numbers" Universal set - the set of all possible values A = {1, 2}B = {3, 4, 5} = {1, 2, 3, 4, 5} Ac or A' Complement - all the elements not in set A = {1, 2, 3, 4, 5}A = {1, 2, 3}Ac = {4,5} [a, b] Closed interval - values between a and b including a and b [1,4] = {1, 2, 3, 4} if only including integers (a, b) Open interval - values between a and b not including a and b (1,4) = {2, 3} if only including integers |A| Order/cardinality - number of elements in the set A = {3, 6, 7, 9}|A| = 4 Natural numbers - only positive numbers with no decimals or fractions {1, 2, 3, ...} Integers - all positive and negative numbers with no decimals or fractions, including 0 {..., -3, -2, -1, 0, 1, 2, 3, ...} Rational numbers - a number that can be represented as a fraction comprised of integers Real numbers - rational numbers and irrational numbers , e, 3, , 0.25 Complex numbers - numbers made up of a real and imaginary component 4 + 2 The concept of a set is one of the most fundamental ideas in mathematics.Essentially, a set is simply a collection of objects. The field of mathematicsthat studies sets, called set theory, was founded by the Germanmathematician GeorgCantor in the latter half of the 19th century. Today the concept of setspermeates almost all of modern mathematics; almost every other mathematicalconcept (including the seemingly fundamental concept of numbers!) has beendefined, directly or indirectly, in terms of sets. Basic definitions and notation A set is a collection of objects, considered as a mathematicalobject in its own right. (A helpful metaphor for a set is a cardboardboxthe box can hold objects, and we can choose to think about theobjects in the box individually, or to think about the box and its contentscollectively as a single object.) The objects in a set are called theelements (or members) of the set; the elements are saidto belong to the set (or to be in the set), and the setis said to contain the elements. Usually the elements of a set areother mathematical objects, such as numbers, variables, or geometricpoints. Writing sets A set is often written by listing its elements between curlybraces{}. For example, a set containing the numbers 1,2,and3 would be written as {1,2,3}. When the elements of a set follow an obvious pattern but there are too manyof them to list explicitly, it is common to list the first few elements (toestablish the pattern) and the last element (to specify where the patternstops), with an ellipsis() in between to indicate that theelements in the middle were omitted. For instance, to write the set of positiveintegers from 1 to100, we can write{1,2,3,,100}. If no element is written afterthe ellipsis, the pattern is assumed to continue forever; so the set written{1,2,3,} contains all of the positive integers.Sometimes the elements of a set go on forever in bothdirectionsfor instance, the set of all integers (bothpositive and negative) can be written as{,3,2,1,0,1,2,3,}. A set can also be described in natural language using English phrases. Forexample, the set of all positive integers describes a particularset. It is important that the description be precise, so that there is no doubtabout whether a particular object is or is not an element of the set. A commonexample of an imprecise description is a phrase such as the numbersbetween 1 and10. This phrase has several ambiguities. Are thenumbers 1 and10 themselves elements of the set, or are they excluded? Areall real numbers between 1 and10 included, or only the integers?Finally, the phrase should probably be clearer about the fact that thesenumbers are to be considered as a set, rather than individually. A more precisedescription of this set might be the set of positive integers no greaterthan10 (if the numbers 1 and10 are in the set, and onlywhole numbers are included) or the set of real numbers between 1and10, exclusive (if the set includes numbers with fractionalparts, but not the numbers 1 and10 themselves). The symbol is used to mean is an element of, justas the symbol= is used to mean equals. For example, to saythat the number2 is an element of the set {1,2,3}, we canwrite 2{1,2,3}. To express the opposite,is not an element of, we put a slash through the symbol andwrite. For example, we can write4{1,2,3}. It is often useful to assign names to sets. These names are usually chosento be single letters, similar to the use of letters to represent numbers inalgebra. A very common convention is to use capital letters for the names ofsets and lower-case letters to represent the elements of sets. So, forinstance, if we assign the letterA to the set{1,2,3} by writing A={1,2,3}, wecan then say that 2A. We can also writeaA, by which we mean thatais a (possibly unknown) number that is an element of thesetAin other words, the value ofa mustbe either 1,2, or3. Equality of sets Two sets are said to be equal if every element of the first setis an element of the second set, and vice versa. For example, ifA={1,2,3} andB={2,3,1}, then the sets AandB are equal. Every element ofA is also anelement ofB, and every element ofB is also anelement ofA. We express set equality with an equals sign, soin this case we write A=B. When two sets areequal, they are considered to be the same set. It is important to remember thatequality between sets is a different concept than equality between numbers. One consequence of this definition of equality is that the order in whichthe elements of a set are listed is irrelevant. We care only about whichobjects are elements of the set, not the order they come in. So, for instance,the expressions {1,2,3} and {2,3,1} both describe thesame set. Another consequence is that the number of times an element is listed isirrelevant. The set {1,2,1,3,3,3} is equal to theset {1,2,3}, because every element of the first set is an elementof the second, and vice versa. (It does not matter that the numbers 1and3 are listed several times in the first set.) [Sometimes we need to use a collection of objects in which order isimportant. Such a collection is called a sequence or anordered list. An example is the use of ordered pairs ofthe form (x,y) to represent points in atwo-dimensional plane; the point(2,5) is different from thepoint(5,2). Similarly, we sometimes need a collection of objects inwhich it is meaningful for an element to appear more than once. Such acollection is called amultiset. Whenwe refer to a set, however, it is to be understood that the order andrepetition of the elements is irrelevant.] Sets containing other sets The elements of a set can be anythingeven other sets. For example,suppose we have two sets, A={1,2,3} andB={2,3,4,5}. Think of A asa box containing the numbers 1,2, and3, and Bas abox containing the numbers 2,3, 4, and5. There is nothing stoppingus from putting the boxA and the boxBtogether inside a larger boxC. In the same way, we can make asetC containing the setA and thesetB as elements. We can write the setC as{A,B}, or, if we like, we can say C={{1,2,3},{2,3,4,5}}. A set containing other sets is like a box containing other boxes. In thiscase, the setC has two elements, which are the twosets A andB. These elements of thesetC are themselves sets; the setA has threeelements, and the setB has four elements. Therefore it is true that AC,because A is an element ofC. Similarly, it is truethat BC. Of course, it is also truethat, say, 1A, because the number1 is anelement of the setA. However, it is not true that 1C,because 1 is not an element of the setC. The only two elementsofC are the sets A andB, andneither of these two elements is the number1. (The setAcontains the number1, but Aitself is not thenumber1.) Returning to the box metaphor, we should think about elements of a set (abox) as the objects that are directly inside the box. Thenumber1 is not directly inside the boxC; instead, it ishidden inside the boxA. So 1is not an element of thesetC, but 1is an element of thesetA. Likewise, even though it is true that 2A andalso true that 2B, it is not true that2C, because the number2 is not directlyinside the boxC. Now lets consider the set D={1,2,3,2,3,4,5}, which is the same as the set {1,2,3,4,5} because theorder and repetition of the elements are irrelevant. The setDis different from the setCthese sets are notequal. To see this, it is enough to find a single element in one set that isnot an element of the other set. Well, the number1, for example, is anelement ofD but (as explained above) is not an elementofC; so the sets are not equal. Thinking of CandD as boxes, we see why they are not the same: thesetC is a box that contains two boxes (which themselves happento contain some numbers), whereas the setD is a box thatdirectly contains five numbers (and no boxes). As another example, consider the two sets E={1} andF={{1}}. These sets are not equal.The setE has one element, which is the number1; thesetF also has one element, but the single elementofF is another set, not a number. So, whereas thesetE is a box containing the number1, thesetF is a box containing a box containing the number1.(In other words, the setF is a boxcontainingEdo you see this?) Both of these aredifferent from the number1 itself. So it is true that1E, and in factEF (because the single elementofF is the setE), but it is not truethat 1F. Sets containing other sets are common in some areas of mathematics, but wewont see them very often in this course. (Mathematicians who work inthese areas often call a set containing other sets a collection ofsets or a family of sets to avoid the awkward phraseset of sets.) Cardinality The cardinality of a set is the number of elements that the setcontains. (Sometimes the cardinality of a set is simply called thesize of the set, because size is ashorter word; but cardinality is the technically correct term.) It is common to write|A| to mean the cardinality of thesetA. For example, ifA={1,4,8}, then|A|=3, because A has three elements. This isthe same notation as is used for the absolute value of a number, but contextmakes it clear what is meant: if the vertical bars surround a set, they referto the cardinality of the set. The cardinality of a set might be infinite. For instance, the cardinality ofthe set of positive integers, {1,2,3,}, isinfinite. The empty set We have been thinking of sets as boxes. What kind of set