I'm not a bot

home / geometryGeometry is a branch of mathematics that includes the study of shape, size, and other properties of figures. It is one of the oldest branches of mathematics and may have been used even in prehistoric times. We can produce geometric figures from real-life situations. Below is a picture where points, lines, and planes represent the walls and structural components of buildings. Geometry is often divided into plane geometry and solid geometry. Plane Geometry Plane geometry includes the study of points, lines, and geometric figures such as circles and polygons that lie in a plane. There are three terms in geometry that are widely used that cannot really be defined using well-defined objects. There is however, a general agreement as to what they mean; that is, the definitions of these terms are axiomatic. They are the basis for creating and working with any geometric figure. Point A point indicates a location in space. It has no dimension and is represented by a dot. Points are typically labeled with capital letters. • Apoint A Line A line has one dimension, its length, and is represented with a line containing arrows indicating it extends in either direction indefinitely. Two non-overlapping points determine a unique line and we can name the line with those two points or any other two points on the line. line AB or BA Plane A plane has two dimensions, length and width, and forms a flat surface (such as a piece of notebook paper) extending in both directions indefinitely. A plane can be named by three noncollinear points or by a letter. Plane ABC or plane P The shapes formed in plane geometry have perimeter and area if they are polygons, circumference and area if they are circles, and length if they are one dimensional, like a line. Solid Geometry Solid geometry is the branch of geometry that studies solid (three-dimensional) figures. The solids are polyhedra (plural for polyhedron) or non-polyhedra. Polyhedron A polyhedron is a solid whose faces are polygons. Octahedron whose faces are triangles Rectangular prism whose faces are rectangles. Non-polyhedron A non-polyhedron is a solid that has all or partly curved surfaces. A right cone has a circular base that tapers to a tip. A sphere's surface is completely round like a ball. The shapes formed in solid geometry have volume and surface area. You can also measure the length for a part of a solid such as the length of an edge of a rectangular prism or the radius of a sphere. Types of Geometry There are numerous branches of geometry that are classified based on various criteria such as the methods of study, the types of objects studied, and more. The following are some of said branches: Analytic Geometry Analytic geometry is the study of plane and solid geometry that uses Algebra and incorporates the two-dimensional coordinate plane or three-dimensional coordinate plane. It is sometimes referred to as coordinate geometry for this reason. Since analytic geometry uses Algebra and the coordinate planes for geometric figures, it tends to be more numerical in nature, and thus has proven to be an important branch of mathematics used by engineers and physicists. Euclidean Geometry Euclidean geometry is the study of plane and solid geometry which uses axioms, postulates, and deductive reasoning to prove theorems about geometric concepts. The kind of geometry most students usually study was recorded in The Elements, a set of books written about 300 BC by Euclid, a Greek mathematician. There are now other kinds of geometry as well. These other kinds originated about 2,000 years after Euclid's work. Non-Euclidean Geometry Non-Euclidean geometry typically uses most of the ideas of Euclidean geometry but uses a different version of Euclid's parallel postulate. In Euclidean geometry two parallel lines never intersect. In Non-Euclidean geometry, parallel lines can intersect depending on which type of geometry is chosen. There are two basic types: Spherical and Hyperbolic Non-Euclidean geometries. Think of folding a plane in Euclidean geometry onto a sphere or a hyperboloid (a three-dimensional hyperbola). The lines are bent and circular or elliptical in spherical geometry, and parabolic or hyperbolic in hyperbolic geometry. Branch of mathematics For other uses, see Geometry (disambiguation). GeometryProjecting a sphere to a plane Branches Euclidean Non-Euclidean Elliptic Spherical Hyperbolic Non-Archimedean geometry Projective Affine Synthetic Analytic Algebraic Arithmetic Diophantine Differential Riemannian Symplectic Discrete differential Complex Finite Discrete/Combinatorial Digital Convex Computational Fractal Incidence Noncommutative geometry Noncommutative algebraic geometry ConceptsFeaturesDimension Straightedge and compass constructions Angle Curve Diagonal Orthogonality (Perpendicular) Parallel Vertex Congruence Similarity Symmetry Zero-dimensional Point One-dimensional Line segment ray Length Two-dimensional Plane Area Polygon Triangle Altitude Hypotenuse Pythagorean theorem Parallelogram Square Rectangle Rhombus Rhomboid Quadrilateral Trapezoid Kite Circle Diameter Circumference Area Three-dimensional Volume Cube cuboid Cylinder Dodecahedron Icosahedron Octahedron Pyramid Platonic Solid Sphere Tetrahedron Four-/other-dimensional Tesseract Hypersphere Geometers by name Aida Aryabhata Ahmes Alhazen Apollonius Archimedes Atiyah Baudhayana Bolyai Brahmagupta Cartan Chern Coxeter Descartes Euclid Euler Gauss Gromov Hilbert Huygens Jyeṣṭhadeva Kātyāyana Khayyám Klein Lobachevsky Manava Minkowski Minggatu Pascal Pythagoras Parameshvara Poincaré Riemann Sakabe Sijzi al-Tusi Veblen Virasena Yang Hui al-Yasamin Zhang List of geometers by period BCE Ahmes Baudhayana Manava Pythagoras Euclid Archimedes Apollonius 1–1400s Zhang Kātyāyana Aryabhata Brahmagupta Virasena Alhazen Sijzi Khayyám al-Yasamin al-Tusi Yang Hui Parameshvara 1400s–1700s Jyeṣṭhadeva Descartes Pascal Huygens Minggatu Euler Sakabe Aida 1700s–1900s Gauss Lobachevsky Bolyai Riemann Klein Poincaré Hilbert Minkowski Cartan Veblen Coxeter Chern Present day Atiyah Gromov vte Part of a series onMathematics History Index Areas Number theory Geometry Algebra Calculus and Analysis Discrete mathematics Logic Set theory Probability Statistics and Decision theory Relationship with sciences Physics Chemistry Geosciences Computation Biology Linguistics Economics Philosophy Education Mathematics Portalvte Geometry (from Ancient Greek γεωμετρία (geōmetría) 'land measurement'; from γῆ (gê) 'earth, land' and μέτρον (métron) 'a measure')[1] is a branch of mathematics concerned with properties of space such as the distance, shape, size, and relative position of figures.[2] Geometry is, along with arithmetic, one of the oldest branches of mathematics. A mathematician who works in the field of geometry is called a geometer. Until the 19th century, geometry was almost exclusively devoted to Euclidean geometry,[a] which includes the notions of point, line, plane, distance, angle, surface, and curve, as fundamental concepts.[3] Originally developed to model the physical world, geometry has applications in almost all sciences, and also in art, architecture, and other activities that are related to graphics.[4] Geometry also has applications in areas of mathematics that are apparently unrelated. For example, methods of algebraic geometry are fundamental in Wiles's proof of Fermat's Last Theorem, a problem that was stated in terms of elementary arithmetic, and remained unsolved for several centuries. During the 19th century several discoveries enlarged dramatically the scope of geometry. One of the oldest such discoveries is Carl Friedrich Gauss's Theorema Egregium ("remarkable theorem") that asserts roughly that the Gaussian curvature of a surface is independent from any specific embedding in a Euclidean space. This implies that surfaces can be studied intrinsically, that is, as stand-alone spaces, and has been expanded into the theory of manifolds and Riemannian geometry. Later in the 19th century, it appeared that geometries without the parallel postulate (non-Euclidean geometries) can be developed without introducing any contradiction. The geometry that underlies general relativity is a famous application of non-Euclidean geometry. Since the late 19th century, the scope of geometry has been greatly expanded, and the field has been split in many subfields that depend on the underlying methods—differential geometry, algebraic geometry, computational geometry, algebraic topology, discrete geometry (also known as combinatorial geometry), etc.—or on the properties of Euclidean spaces that are disregarded—projective geometry that consider only alignment of points but not distance and parallelism, affine geometry that omits the concept of angle and distance, finite geometry that omits continuity, and others. This enlargement of the scope of geometry led to a change of meaning of the word "space", which originally referred to the three-dimensional space of the physical world and its model provided by Euclidean geometry; presently a geometric space, or simply a space is a mathematical structure on which some geometry is defined. Main article: History of geometry A European and an Arab practicing geometry in the 15th century The earliest recorded beginnings of geometry can be traced to ancient Mesopotamia and Egypt in the 2nd millennium BC.[5][6] Early geometry was a collection of empirically discovered principles concerning lengths, angles, areas, and volumes, which were developed to meet some practical need in surveying, construction, astronomy, and various crafts. The earliest known texts on geometry are the Egyptian Rhind Papyrus (2000–1800 BC) and Moscow Papyrus (c. 1890 BC), and the Babylonian clay tablets, such as Plimpton 322 (1900 BC). For example, the Moscow Papyrus gives a formula for calculating the volume of a truncated pyramid, or frustum.[7] Later clay tablets (350–50 BC) demonstrate that Babylonian astronomers implemented trapezoid procedures for computing Jupiter's position and motion within time-velocity space. These geometric procedures anticipated the Oxford Calculators, including the mean speed theorem, by 14 centuries.[8] South of Egypt the ancient Nubians established a system of geometry including early versions of sun clocks.[9][10] In the 7th century BC, the Greek mathematician Thales of Miletus used geometry to solve problems such as calculating the height of pyramids and the distance of ships from the shore. He is credited with the first use of deductive reasoning applied to geometry, by deriving four corollaries to Thales's theorem.[11] Pythagoras established the Pythagorean School, which is credited with the first proof of the Pythagorean theorem,[12] though the statement of the theorem has a long history.[13][14] Eudoxus (408–c. 355 BC) developed the method of exhaustion, which allowed the calculation of areas and volumes of curvilinear figures,[15] as well as a theory of ratios that avoided the problem of incommensurable magnitudes, which enabled subsequent geometers to make significant advances. Around 300 BC, geometry was revolutionized by Euclid, whose Elements, widely considered the most successful and influential textbook of all time,[16] introduced mathematical rigor through the axiomatic method and is the earliest example of the format still used in mathematics today, that of definition, axiom, theorem, and proof. Although most of the contents of the Elements were already known, Euclid arranged them into a single, coherent logical framework.[17] The Elements was known to all educated people in the West until the middle of the 20th century and its contents are still taught in geometry classes today.[18] Archimedes (c. 287–212 BC) of Syracuse, Italy used the method of exhaustion to calculate the area under the arc of a parabola with the summation of an infinite series, and gave remarkably accurate approximations of pi.[19] He also studied the spiral bearing his name and obtained formulas for the volumes of surfaces of revolution. Woman teaching geometry. Illustration at the beginning of a medieval translation of Euclid's Elements, (c. 1310). Indian mathematicians also made many important contributions in geometry. The Shatapatha Brahmana (3rd century BC) contains rules for ritual geometric constructions that are similar to the Sulba Sutras.[20] According to (Hayashi 2005, p. 363), the Śulba Sūtras contain "the earliest extant verbal expression of the Pythagorean Theorem in the world, although it had already been known to the Old Babylonians. They contain lists of Pythagorean triples,[b] which are particular cases of Diophantine equations.[21] In the Bakhshali manuscript, there are a handful of geometric problems (including problems about volumes of irregular solids). The Bakhshali manuscript also "employs a decimal place value system with a dot for zero."[22] Aryabhata's Aryabhatiya (499) includes the computation of areas and volumes. Brahmagupta wrote his astronomical work Brāhmasphuṭasiddhānta in 628. Chapter 12, containing 66 Sanskrit verses, was divided into two sections: "basic operations" (including cube roots, fractions, ratio and proportion, and barter) and "practical mathematics" (including mixture, mathematical series, plane figures, stacking bricks, sawing of timber, and piling of grain).[23] In the latter section, he stated his famous theorem on the diagonals of a cyclic quadrilateral. Chapter 12 also included a formula for the area of a cyclic quadrilateral (a generalization of Heron's formula), as well as a complete description of rational triangles (i.e. triangles with rational sides and rational areas).[23] In the Middle Ages, mathematics in medieval Islam contributed to the development of geometry, especially algebraic geometry.[24][25] Al-Mahani (b. 853) conceived the idea of reducing geometrical problems such as duplicating the cube to problems in algebra.[26] Thābit ibn Qurra (known as Thebit in Latin) (836–901) dealt with arithmetic operations applied to ratios of geometrical quantities, and contributed to the development of analytic geometry.[27] Omar Khayyam (1048–1131) found geometric solutions to cubic equations.[28] The theorems of Ibn al-Haytham (Alhazen), Omar Khayyam and Nasir al-Din al-Tusi on quadrilaterals, including the Lambert quadrilateral and Saccheri quadrilateral, were part of a line of research on the parallel postulate continued by later European geometers, including Vitello (c. 1230 – c. 1314), Gersonides (1288–1344), Alfonso, John Wallis, and Giovanni Girolamo Saccheri, that by the 19th century led to the discovery of hyperbolic geometry.[29] In the early 17th century, there were two important developments in geometry. The first was the creation of analytic geometry, or geometry with coordinates and equations, by René Descartes (1596–1650) and Pierre de Fermat (1601–1665).[30] This was a necessary precursor to the development of calculus and a precise quantitative science of physics.[31] The second geometric development of this period was the systematic study of projective geometry by Girard Desargues (1591–1661).[32] Projective geometry studies properties of shapes which are unchanged under projections and sections, especially as they relate to artistic perspective.[33] Two developments in geometry in the 19th century changed the way it had been studied previously.[34] These were the discovery of non-Euclidean geometries by Nikolai Ivanovich Lobachevsky, János Bolyai and Carl Friedrich Gauss and of the formulation of symmetry as the central consideration in the Erlangen programme of Felix Klein (which generalized the Euclidean and non-Euclidean geometries). Two of the master geometers of the time were Bernhard Riemann (1826–1866), working primarily with tools from mathematical analysis, and introducing the Riemann surface, and Henri Poincaré, the founder of algebraic topology and the geometric theory of dynamical systems. As a consequence of these major changes in the conception of geometry, the concept of "space" became something rich and varied, and the natural background for theories as different as complex analysis and classical mechanics.[35] The following are some of the most important concepts in geometry.[3][36] An illustration of Euclid's parallel postulate See also: Euclidean geometry and Axiom Euclid took an abstract approach to geometry in his Elements,[37] one of the most influential books ever written.[38] Euclid introduced certain axioms, or postulates, expressing primary or self-evident properties of points, lines, and planes.[39] He proceeded to rigorously deduce other properties by mathematical reasoning. The characteristic feature of Euclid's approach to geometry was its rigor, and it has come to be known as axiomatic or synthetic geometry.[40] At the start of the 19th century, the discovery of non-Euclidean geometries by Nikolai Ivanovich Lobachevsky (1792–1856), János Bolyai (1802–1860), Carl Friedrich Gauss (1777–1855) and others[41] led to a revival of interest in this discipline, and in the 20th century, David Hilbert (1862–1943) employed axiomatic reasoning in an attempt to provide a modern foundation of geometry.[42] Main article: Point (geometry) Points are generally considered fundamental objects for building geometry. They may be defined by the properties that they must have, as in Euclid's definition as "that which has no part",[43] or in synthetic geometry. In modern mathematics, they are generally defined as elements of a set called space, which is itself axiomatically defined. With these modern definitions, every geometric shape is defined as a set of points; this is not the case in synthetic geometry, where a line is another fundamental object that is not viewed as the set of the points through which it passes. However, there are modern geometries in which points are not primitive objects, or even without points.[44][45] One of the oldest such geometries is Whitehead's point-free geometry, formulated by Alfred North Whitehead in 1919–1920. Main article: Line (geometry) Euclid described a line as "breadthless length" which "lies equally with respect to the points on itself".[43] In modern mathematics, given the multitude of geometries, the concept of a line is closely tied to the way the geometry is described. For instance, in analytic geometry, a line in the plane is often defined as the set of points whose coordinates satisfy a given linear equation,[46] but in a more abstract setting, such as incidence geometry, a line may be an independent object, distinct from the set of points which lie on it.[47] In differential geometry, a geodesic is a generalization of the notion of a line to curved spaces.[48] Main article: Euclidean plane In Euclidean geometry a plane is a flat, two-dimensional surface that extends infinitely;[43] the definitions for other types of geometries are generalizations of that. Planes are used in many areas of geometry. For instance, planes can be studied as a topological surface without reference to distances or angles;[49] it can be studied as an affine space, where collinearity and ratios can be studied but not distances;[50] it can be studied as the complex plane using techniques of complex analysis;[51] and so on. Main article: Curve (geometry) A curve is a 1-dimensional object that may be straight (like a line) or not; curves in 2-dimensional space are called plane curves and those in 3-dimensional space are called space curves.[52] In topology, a curve is defined by a function from an interval of the real numbers to another space.[49] In differential geometry, the same definition is used, but the defining function is required to be differentiable.[53] Algebraic geometry studies algebraic curves, which are defined as algebraic varieties of dimension one.[54] Main article: Surface (mathematics) A sphere is a surface that can be defined parametrically (by x = r sin θ cos φ, y = r sin θ sin φ, z = r cos θ) or implicitly (by x2 + y2 + z2 − r2 = 0). A surface is a two-dimensional object, such as a sphere or paraboloid.[55] In differential geometry[53] and topology,[49] surfaces are described by two-dimensional 'patches' (or neighborhoods) that are assembled by diffeomorphisms or homeomorphisms, respectively. In algebraic geometry, surfaces are described by polynomial equations.[54] Main article: Solid geometry In Euclidean space, a ball is the volume bounded by a sphere. A solid is a three-dimensional object bounded by a closed surface; for example, a ball is the volume bounded by a sphere. Main article: Manifold A manifold is a generalization of the concepts of curve and surface. In topology, a manifold is a topological space where every point has a neighborhood that is homeomorphic to Euclidean space.[49] In differential geometry, a differentiable manifold is a space where each neighborhood is diffeomorphic to Euclidean space.[53] Manifolds are used extensively in physics, including in general relativity and string theory.[56] Main article: Angle Acute (a), obtuse (b), and straight (c) angles. The acute and obtuse angles are also known as oblique angles. Euclid defines a plane angle as the inclination to each other, in a plane, of two lines which meet each other, and do not lie straight with respect to each other.[43] In modern terms, an angle is the figure formed by two rays, called the sides of the angle, sharing a common endpoint, called the vertex of the angle.[57] The size of an angle is formalized as an angular measure. In Euclidean geometry, angles are used to study polygons and triangles, as well as forming an object of study in their own right.[43] The study of the angles of a triangle or of angles in a unit circle forms the basis of trigonometry.[58] In differential geometry and calculus, the angles between plane curves or space curves or surfaces can be calculated using the derivative.[59][60] Main articles: Length, Area, and Volume See also: Area § List of formulas, and Volume § Volume formulas Length, area, and volume describe the size or extent of an object in one dimension, two dimension, and three dimensions respectively. In Euclidean geometry and analytic geometry, the length of a line segment can often be calculated by the Pythagorean theorem.[61] Area and volume can be defined as fundamental quantities separate from length, or they can be described and calculated in terms of lengths in a plane or 3-dimensional space. Mathematicians have found many explicit formulas for area and formulas for volume of various geometric objects. In calculus, area and volume can be defined in terms of integrals, such as the Riemann integral[62] or the Lebesgue integral.[63] Other geometrical measures include the curvature and compactness. Main articles: Metric (mathematics) and Measure (mathematics) Visual checking of the Pythagorean theorem for the (3, 4, 5) triangle as in the Zhoubi Suanjing 500–200 BC. The Pythagorean theorem is a consequence of the Euclidean metric. The concept of length or distance can be generalized, leading to the idea of metrics.[64] For instance, the Euclidean metric measures the distance between points in the Euclidean plane, while the hyperbolic metric measures the distance in the hyperbolic plane. Other important examples of metrics include the Lorentz metric of special relativity and the semi-Riemannian metrics of general relativity.[65] In a different direction, the concepts of length, area and volume are extended by measure theory, which studies methods of assigning a size or measure to sets, where the measures follow rules similar to those of classical area and volume.[66] Main articles: Congruence (geometry) and Similarity (geometry) Congruence and similarity are concepts that describe when two shapes have similar characteristics.[67] In Euclidean geometry, similarity is used to describe objects that have the same shape, while congruence is used to describe objects that are the same in both size and shape.[68] Hilbert, in his work on creating a more rigorous foundation for geometry, treated congruence as an undefined term whose properties are defined by axioms. Congruence and similarity are generalized in transformation geometry, which studies the properties of geometric objects that are preserved by different kinds of transformations.[69] Main article: Compass and straightedge constructions Classical geometers paid special attention to constructing geometric objects that had been described in some other way. Classically, the only instruments used in most geometric constructions are the compass and straightedge.[c] Also, every construction had to be complete in a finite number of steps. However, some problems turned out to be difficult or impossible to solve by these means alone, and ingenious constructions using neusis, parabolas and other curves, or mechanical devices, were found. Main articles: Rotation (geometry) and Orientation (geometry) The geometrical concepts of rotation and orientation define part of the placement of objects embedded in the plane or in space. For broader coverage of this topic, see Dimension (mathematics). The Koch snowflake, with fractal dimension=log4/log3 and topological dimension=1 Traditional geometry allowed dimensions 1 (a line or curve), 2 (a plane or surface), and 3 (our ambient world conceived of as three-dimensional space). Furthermore, mathematicians and physicists have used higher dimensions for nearly two centuries.[70] One example of a mathematical use for higher dimensions is the configuration space of a physical system, which has a dimension equal to the system's degrees of freedom. For instance, the configuration of a screw can be described by five coordinates.[71] In general topology, the concept of dimension has been extended from natural numbers, to infinite dimension (Hilbert spaces, for example) and positive real numbers (in fractal geometry).[72] In algebraic geometry, the dimension of an algebraic variety has received a number of apparently different definitions, which are all equivalent in the most common cases.[73] Main article: Symmetry A tiling of the hyperbolic plane The theme of symmetry in geometry is nearly as old as the science of geometry itself.[74] Symmetric shapes such as the circle, regular polygons and platonic solids held deep significance for many ancient philosophers[75] and were investigated in detail before the time of Euclid.[39] Symmetric patterns occur in nature and were artistically rendered in a multitude of forms, including the graphics of Leonardo da Vinci, M. C. Escher, and others.[76] In the second half of the 19th century, the relationship between symmetry and geometry came under intense scrutiny. Felix Klein's Erlangen program proclaimed that, in a very precise sense, symmetry, expressed via the notion of a transformation group, determines what geometry is.[77] Symmetry in classical Euclidean geometry is represented by congruences and rigid motions, whereas in projective geometry an analogous role is played by collineations, geometric transformations that take straight lines into straight lines.[78] However it was in the new geometries of Bolyai and Lobachevsky, Riemann, Clifford and Klein, and Sophus Lie that Klein's idea to 'define a geometry via its symmetry group' found its inspiration.[79] Both discrete and continuous symmetries play prominent roles in geometry, the former in topology and geometric group theory,[80][81] the latter in Lie theory and Riemannian geometry.[82][83] A different type of symmetry is the principle of duality in projective geometry, among other fields. This meta-phenomenon can roughly be described as follows: in any theorem, exchange point with plane, join with meet, lies in with contains, and the result is an equally true theorem.[84] A similar and closely related form of duality exists between a vector space and its dual space.[85] Main article: Euclidean geometry Euclidean geometry is geometry in its classical sense.[86] As it models the space of the physical world, it is used in many scientific areas, such as mechanics, astronomy, crystallography,[87] and many technical fields, such as engineering,[88] architecture,[89] geodesy,[90] aerodynamics,[91] and navigation.[92] The mandatory educational curriculum of the majority of nations includes the study of Euclidean concepts such as points, lines, planes, angles, triangles, congruence, similarity, solid figures, circles, and analytic geometry.[93] Main article: Euclidean vector Euclidean vectors are used for a myriad of applications in physics and engineering, such as position, displacement, deformation, velocity, acceleration, force, etc. Differential geometry uses tools from calculus to study problems involving curvature. Main article: Differential geometry Differential geometry uses techniques of calculus and linear algebra to study problems in geometry.[94] It has applications in physics,[95] econometrics,[96] and bioinformatics,[97] among others. In particular, differential geometry is of importance to mathematical physics due to Albert Einstein's general relativity postulation that the universe is curved.[98] Differential geometry can either be intrinsic (meaning that the spaces it considers are smooth manifolds whose geometric structure is governed by a Riemannian metric, which determines how distances are measured near each point) or extrinsic (where the object under study is a part of some ambient flat Euclidean space).[99] This section is an excerpt from Non-Euclidean geometry.[edit] Behavior of lines with a common perpendicular in each of the three types of geometry In mathematics, non-Euclidean geometry consists of two geometries based on axioms closely related to those that specify Euclidean geometry. As Euclidean geometry lies at the intersection of metric geometry and affine geometry, non-Euclidean geometry arises by either replacing the parallel postulate with an alternative, or relaxing the metric requirement. In the former case, one obtains hyperbolic geometry and elliptic geometry, the traditional non-Euclidean geometries. When the metric requirement is relaxed, then there are affine planes associated with the planar algebras, which give rise to kinematic geometries that have also been called non-Euclidean geometry. Main article: Topology A thickening of the trefoil knot Topology is the field concerned with the properties of continuous mappings,[100] and can be considered a generalization of Euclidean geometry.[101] In practice, topology often means dealing with large-scale properties of spaces, such as connectedness and compactness.[49] The field of topology, which saw massive development in the 20th century, is in a technical sense a type of transformation geometry, in which transformations are homeomorphisms.[102] This has often been expressed in the form of the saying 'topology is rubber-sheet geometry'. Subfields of topology include geometric topology, differential topology, algebraic topology and general topology.[103] Main article: Algebraic geometry Quintic Calabi–Yau threefold Algebraic geometry is fundamentally the study by means of algebraic methods of some geometrical shapes, called algebraic sets, and defined as common zeros of multivariate polynomials.[104] Algebraic geometry became an autonomous subfield of geometry c. 1900, with a theorem called Hilbert's Nullstellensatz that establishes a strong correspondence between algebraic sets and ideals of polynomial rings. This led to a parallel development of algebraic geometry, and its algebraic counterpart, called commutative algebra.[105] From the late 1950s through the mid-1970s algebraic geometry had undergone major foundational development, with the introduction by Alexander Grothendieck of scheme theory, which allows using topological methods, including cohomology theories in a purely algebraic context.[105] Scheme theory allowed to solve many difficult problems not only in geometry, but also in number theory. Wiles' proof of Fermat's Last Theorem is a famous example of a long-standing problem of number theory whose solution uses scheme theory and its extensions such as stack theory. One of seven Millennium Prize problems, the Hodge conjecture, is a question in algebraic geometry.[106] Algebraic geometry has applications in many areas, including cryptography[107] and string theory.[108] Main article: Complex geometry Complex geometry studies the nature of geometric structures modelled on, or arising out of, the complex plane.[109][110][111] Complex geometry lies at the intersection of differential geometry, algebraic geometry, and analysis of several complex variables, and has found applications to string theory and mirror symmetry.[112] Complex geometry first appeared as a distinct area of study in the work of Bernhard Riemann in his study of Riemann surfaces.[113][114][115] Work in the spirit of Riemann was carried out by the Italian school of algebraic geometry in the early 1900s. Contemporary treatment of complex geometry began with the work of Jean-Pierre Serre, who introduced the concept of sheaves to the subject, and illuminated the relations between complex geometry and algebraic geometry.[116][117] The primary objects of study in complex geometry are complex manifolds, complex algebraic varieties, and complex analytic varieties, and holomorphic vector bundles and coherent sheaves over these spaces. Special examples of spaces studied in complex geometry include Riemann surfaces, and Calabi–Yau manifolds, and these spaces find uses in string theory. In particular, worldsheets of strings are modelled by Riemann surfaces, and superstring theory predicts that the extra 6 dimensions of 10 dimensional spacetime may be modelled by Calabi–Yau manifolds. Main article: Discrete geometry Discrete geometry includes the study of various sphere packings. Discrete geometry is a subject that has close connections with convex geometry.[118][119][120] It is concerned mainly with questions of relative position of simple geometric objects, such as points, lines and circles. Examples include the study of sphere packings, triangulations, the Kneser-Poulsen conjecture, etc.[121][122] It shares many methods and principles with combinatorics. Main article: Computational geometry Computational geometry deals with algorithms and their implementations for manipulating geometrical objects. Important problems historically have included the travelling salesman problem, minimum spanning trees, hidden-line removal, and linear programming.[123] Although being a young area of geometry, it has many applications in computer vision, image processing, computer-aided design, medical imaging, etc.[124] Main article: Geometric group theory The Cayley graph of the free group on two generators a and b Groups have been understood as geometric objects since Klein's Erlangen programme. Geometric group theory studies group actions on objects that are regarded as geometric (significantly, isometric actions on metric spaces) to study finitely generated groups, often involving large-scale geometric techniques[125] and borrowing from topology, geometry, dynamics and analysis.[126] It had a significant impact on low-dimensional topology, a celebrated result being Agol's proof of the virtually Haken conjecture that combines Perelman geometrization with cubulation techniques.[127] Group actions on their Cayley graphs are foundational examples of isometric group actions. Other major topics include quasi-isometries, Gromov-hyperbolic groups and their generalizations (relatively and acylindrically hyperbolic groups), free groups and their automorphisms, groups acting on trees, various notions of nonpositive curvature for groups (CAT(0) groups, Dehn functions, automaticity...), right angled Artin groups, and topics close to combinatorial group theory such as small cancellation theory and algorithmic problems (e.g. the word, conjugacy, and isomorphism problems). Other group-theoretic topics like mapping class groups, property (T), solvability, amenability and lattices in Lie groups are sometimes regarded as strongly geometric as well.[125][128][129][130] Main article: Convex geometry Convex geometry investigates convex shapes in the Euclidean space and its more abstract analogues, often using techniques of real analysis and discrete mathematics.[131] It has close connections to convex analysis, optimization and functional analysis and important applications in number theory. Convex geometry dates back to antiquity.[131] Archimedes gave the first known precise definition of convexity. The isoperimetric problem, a recurring concept in convex geometry, was studied by the Greeks as well, including Zenodorus. Archimedes, Plato, Euclid, and later Kepler and Coxeter all studied convex polytopes and their properties. From the 19th century on, mathematicians have studied other areas of convex mathematics, including higher-dimensional polytopes, volume and surface area of convex bodies, Gaussian curvature, algorithms, tilings and lattices. Geometry has found applications in many fields, some of which are described below. Main article: Mathematics and art Bou Inania Madrasa, Fes, Morocco, zellige mosaic tiles forming elaborate geometric tessellations Mathematics and art are related in a variety of ways. For instance, the theory of perspective showed that there is more to geometry than just the metric properties of figures: perspective is the origin of projective geometry.[132] Artists have long used concepts of proportion in design. Vitruvius developed a complicated theory of ideal proportions for the human figure.[133] These concepts have been used and adapted by artists from Michelangelo to modern comic book artists.[134] The golden ratio is a particular proportion that has had a controversial role in art. Often claimed to be the most aesthetically pleasing ratio of lengths, it is frequently stated to be incorporated into famous works of art, though the most reliable and unambiguous examples were made deliberately by artists aware of this legend.[135] Tilings, or tessellations, have been used in art throughout history. Islamic art makes frequent use of tessellations, as did the art of M. C. Escher.[136] Escher's work also made use of hyperbolic geometry. Cézanne advanced the theory that all images can be built up from the sphere, the cone, and the cylinder. This is still used in art theory today, although the exact list of shapes varies from author to author.[137][138] Main articles: Mathematics and architecture and Architectural geometry Geometry has many applications in architecture. In fact, it has been said that geometry lies at the core of architectural design.[139][140] Applications of geometry to architecture include the use of projective geometry to create forced perspective,[141] the use of conic sections in constructing domes and similar objects,[89] the use of tessellations,[89] and the use of symmetry.[89] Main article: Mathematical physics The field of astronomy, especially as it relates to mapping the positions of stars and planets on the celestial sphere and describing the relationship between movements of celestial bodies, have served as an important source of geometric problems throughout history.[142] Riemannian geometry and pseudo-Riemannian geometry are used in general relativity.[143] String theory makes use of several variants of geometry,[144] as does quantum information theory.[145] The Pythagoreans discovered that the sides of a triangle could have incommensurable lengths. Calculus was strongly influenced by geometry.[30] For instance, the introduction of coordinates by René Descartes and the concurrent developments of algebra marked a new stage for geometry, since geometric figures such as plane curves could now be represented analytically in the form of functions and equations. This played a key role in the emergence of infinitesimal calculus in the 17th century. Analytic geometry continues to be a mainstay of pre-calculus and calculus curriculum.[146][147] Another important area of application is number theory.[148] In ancient Greece the Pythagoreans considered the role of numbers in geometry. However, the discovery of incommensurable lengths contradicted their philosophical views.[149] Since the 19th century, geometry has been used for solving problems in number theory, for example through the geometry of numbers or, more recently, scheme theory, which is used in Wiles's proof of Fermat's Last Theorem.[150] Mathematics portal Main category: Geometry Lists List of geometers Category:Algebraic geometers Category:Differential geometers Category:Geometers Category:Topologists List of formulas in elementary geometry List of geometry topics List of important publications in geometry Lists of mathematics topics Related topics Descriptive geometry Flatland, a book written by Edwin Abbott Abbott about two- and three-dimensional space, to understand the concept of four dimensions List of interactive geometry software Other applications Molecular geometry ^ Until the 19th century, geometry was dominated by the assumption that all geometric constructions were Euclidean. In the 19th century and later, this was challenged by the development of hyperbolic geometry by Lobachevsky and other non-Euclidean geometries by Gauss and others. It was then realised that implicitly non-Euclidean geometry had appeared throughout history, including the work of Desargues in the 17th century, all the way back to the implicit use of spherical geometry to understand the Earth's geodesy and to navigate the oceans since antiquity. ^ Pythagorean triples are triples of integers (a , b , c) {\displaystyle (a,b,c)} with the property: a 2 + b 2 = c 2 {\displaystyle a^{2}+b^{2}=c^{2}} . Thus, 3 2 + 4 2 = 5 2 {\displaystyle 3^{2}+4^{2}=5^{2}} , 8 2 + 15 2 = 17 2 {\displaystyle 8^{2}+15^{2}=17^{2}} , 12 2 + 35 2 = 37 2 {\displaystyle 12^{2}+35^{2}=37^{2}} etc. ^ The ancient Greeks had some constructions using other instruments. ^ "Geometry - Formulas, Examples | Plane and Solid Geometry". Cuemath. Retrieved 31 August 2023. ^ Vincenzo De Risi (2015). 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Modular Forms and Fermat's Last Theorem. Springer Science & Business Media. ISBN 978-1-4612-1974-3. Archived from the original on 30 December 2019. Retrieved 25 September 2019. Boyer, C.B. (1991) [1989]. A History of Mathematics (Second edition, revised by Uta C. Merzbach ed.). New York: Wiley. ISBN 978-0-471-54397-8. Cooke, Roger (2005). The History of Mathematics. New York: Wiley-Interscience. ISBN 978-0-471-44459-6. Hayashi, Takao (2003). "Indian Mathematics". In Grattan-Guinness, Ivor (ed.). Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences. Vol. 1. Baltimore, MD: The Johns Hopkins University Press. pp. 118–130. ISBN 978-0-8018-7396-6. Hayashi, Takao (2005). "Indian Mathematics". In Flood, Gavin (ed.). The Blackwell Companion to Hinduism. Oxford: Basil Blackwell. pp. 360–375. ISBN 978-1-4051-3251-0. Jay Kappraff (2014). A Participatory Approach to Modern Geometry. World Scientific Publishing. doi:10.1142/8952. ISBN 978-981-4556-70-5. Zbl 1364.00004. Nikolai I. Lobachevsky (2010). Pangeometry. Heritage of European Mathematics Series. Vol. 4. translator and editor: A. Papadopoulos. European Mathematical Society. Leonard Mlodinow (2002). Euclid's Window – The Story of Geometry from Parallel Lines to Hyperspace (UK ed.). Allen Lane. ISBN 978-0-7139-9634-0. Geometry at Wikipedia's sister projects Definitions from WiktionaryMedia from CommonsNews from WikinewsQuotations from WikiquoteTexts from WikisourceTextbooks from WikibooksResources from Wikiversity Wikibooks has more on the topic of: Geometry Library resources about Geometry Resources in your library "Geometry" . Encyclopædia Britannica. Vol. 11 (11th ed.). 1911. pp. 675–736. A geometry course from Wikiversity Unusual Geometry Problems The Math Forum – Geometry The Math Forum – K–12 Geometry The Math Forum – College Geometry The Math Forum – Advanced Geometry Nature Precedings – Pegs and Ropes Geometry at Stonehenge The Mathematical Atlas – Geometric Areas of Mathematics "4000 Years of Geometry", lecture by Robin Wilson given at Gresham College, 3 October 2007 (available for MP3 and MP4 download as well as a text file) Finitism in Geometry at the Stanford Encyclopedia of Philosophy The Geometry Junkyard Interactive geometry reference with hundreds of applets Dynamic Geometry Sketches (with some Student Explorations) Geometry classes at Khan Academy Retrieved from " Geometry is the field of mathematics dealing with figures in a given space. It is one of the two oldest branches of mathematics, along with arithmetic (which eventually branched into number theory and algebra). The geometry usually studied is Introductory Videos \\ Euclidean Geometry Main article: Euclidean geometry The most common type of geometry used in pre-collegiate mathematics competitions is Euclidean geometry. This type of geometry was first formally outlined by the Greek mathematician Euclid in his book The Elements. Parallel Postulate Main article: Parallel Postulate The fifth postulate stated in the book, is the following statement: If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles. Though some resources state: “Through any line and a point not on the line, there is exactly one line passing through that point parallel to the line” This postulate was the subject of a controversy for many centuries, with many attempted proofs. It is much less simple than the other postulates, and more wordy. This postulate is the basis of Euclidean geometry. There are also other types of geometry that don’t follow this postulate, like hyperbolic geometry and spherical geometry, which say that there are more than 2 parallel lines, or there are no parallel lines, respectively. Non-Euclidean Geometry Non-Euclidean geometry are geometries in which the fifth postulate is altered. Types of non-Euclidean geometry include: Elliptical geometry Hyperbolic geometry Student Guides to Geometry Main Concepts The notion of dimensions is fundamental to geometry. N-space is a term related to this concept. A point is a geometric structure with no area, length, width, or dimension. Its only property is space. It is said to be zero-dimensional. A line is generally taken to mean a straight line, which is the locus of points on the Cartesian plane satisfying a linear function. It has length and position, but no other properties. It is one-dimensional. A line segment means a finite segment of a line, while a ray is a line infinitely extending in only one direction. A plane is a line but in a Cartesian space. It as length, width, and position. It is two-dimensional. The point/line/plane sequence can be extended to spaces and higher dimensions. An angle is a structure formed by the intersection two rays at their endpoints. It is measure in either degrees or radians, though the less-common metric unit gradian is also used. See Also Point The Elements Topology Geometry is the branch of mathematics that deals with the study of points, lines, angles, surfaces, and solids. Understanding these fundamental concepts lays the foundation for exploring more advanced topics in geometry. This lesson introduces these basic geometric elements with definitions, examples, and visual aids to enhance your comprehension. Definition: A point is a precise location in space. It has no size, dimension, or area—only position. It is represented by a dot.Notation: Points are usually named with uppercase letters (e.g., Point A).Real-Life Example: A specific location on a map, such as the location of a city. A Definition: A line is a collection of points that extends infinitely in both directions. It has length but no width or thickness. The two arrows are used to show that it extends forever.Notation: Lines are named using two points on the line (e.g., Line AF or Line FA).Real-Life Example: A straight road that extends endlessly in both directions. A F We put two points in order to name the line as line AF. However, there are an infinite amount of points. You can also name it line FA. Definition: A line segment is a part of a line that has two distinct endpoints. A segment has two endpoints while a line has no endpoints.Example: Segment AF has endpoints at Point A and Point F.Real-Life Example: A specific section of a road between two intersections. A F 4. Ray Definition: A ray starts at one point (the endpoint) and extends infinitely in one direction.Example: Ray AF starts at Point A and passes through Point F, extending indefinitely beyond Point F.Real-Life Example: A laser beam that starts at a specific point and continues indefinitely in one direction. A F 5. Angle Definition: An angle is formed by two rays that share the same endpoint, called the vertex.Example: Angle FAC is formed by Ray FA and Ray AC with Point A as the vertex.Real-Life Example: The corner of a book or the intersection where two streets meet. A F C Definition: A plane is a flat, two-dimensional surface that extends infinitely in all directions. We can use arrows to show that it extends in all directions forever. A plane has length and width but no thickness.Notation: Planes are often named using three non-collinear points (e.g., Plane ABC) or represented with a symbol.Real-Life Example: The surface of a tabletop or a perfectly flat sheet of paper. 7. Parallel Lines Definition: Parallel lines are two lines in the same plane that never meet, no matter how far they are extended.Example: Lines m and n are parallel because they never intersect.Real-Life Example: Railroad tracks that run side by side without ever meeting. m n 8. Intersecting Lines Definition: Intersecting lines are lines that meet or cross each other at a single point.Example: Lines p and q intersect at Point X.Real-Life Example: Crossroads where two roads meet. p q X 9. Vertex Definition: The vertex is the common endpoint where two rays meet to form an angle.Example: Below and in blue you can see what the vertex looks like for an angle, a cube, intersecting lines, and a triangle.Real-Life Example: The corner of a room where two walls meet. Intersecting Lines V Angle V Triangle Vertex V Cube Vertex V Understanding these basic geometric concepts is essential for delving deeper into the study of geometry. These elements form the building blocks for more complex shapes, theorems, and proofs. Practice identifying and drawing these geometric entities to strengthen your grasp of the subject. 1. Lines and angles 13. Transformations Geometry is a branch of mathematics that studies the sizes, shapes, positions, angles, and dimensions of things. Flat shapes like squares, circles, and triangles are a part of flat geometry and are called 2D shapes. These shapes have only 2 dimensions, the length and the width. Examples of 2D shapes in flat geometry are as shown below. 2D shapes can be further classified as open shapes and closed shapes. Open shapes can be defined as a shape or figure whose line segments and/or curves do not meet. They do not start and end at the same point. Closed shapes are geometric shapes that begin and end at the same point. In geometry, a three-dimensional shape can be defined as a solid figure or an object or shape that has three dimensions – length, width, and height. Unlike two-dimensional shapes, three-dimensional shapes have thickness or depth. The attributes of a three-dimensional figure are faces, edges, and vertices. The three dimensions compose the edges of a 3D geometric shape. A cube, rectangular prism, sphere, cone and cylinder are the basic 3-dimensional shapes we see around us. In geometry, an angle can be defined as the figure formed by two rays meeting at a common endpoint. An angle is represented by the symbol ∠. Angles are measured in degrees (°) using a protractor. For example, 45 degrees is represented as 45°. The angles are classified based on their measurements as: An acute angle measures less than 90°. An obtuse angle is between 90° and 180°. A right angle precisely measures 90°. An angle measuring exactly 180° is a straight angle. A reflex angle measures between 180°- 360°. A complete angle measures 360°. The vertex of a shape where two edges meet form an angle. Different shapes in geometry have different angle measures. For example: A triangle is a 3 sided shape, and the sum of its 3 interior angles is 180˚ A square, rectangle or quadrilateral are 4 sided shapes, and the sum of their 4 interior angles is 360˚ Other polygons like the pentagon, hexagon, heptagon, octagon have 5, 6, 7, 8 sides respectively and varying angles. Examples of different Polygons with their angles and sides are as shown below. We learn various aspects of shapes, like the measurement of angles, length of sides, area, volume, etc in geometry. Similarity and congruence are two important aspects of geometry. Similarity: Similarity is when two shapes are the same but their sizes may vary. Congruence: Congruence is when two shapes are exactly the same in shape and size. A coordinate plane is a 2D surface formed by using two number lines that intersect each other at the right angle. The horizontal number line is the x-axis, and the vertical number line is the y-axis. The intersection of the two axes is the (0,0) coordinate. Using the coordinate plane, we plot points, lines, etc. By joining various points on the coordinate plane, we can create shapes. We use Formula and Theorems to solve the geometry problems. A formula is a mathematical equation to solve a geometry problem while a theorem is a statement that is proved using previously known facts. For example, the “Pythagoras Theorem” proved that a2 + b2 = c2 for a right-angled triangle, where a and b are the sides of the right-angled triangle, and c is the hypotenuse. However, a2 + b2 = c2 is the formula for finding the hypotenuse of a right-angled triangle. – The word geometry is made from the Greek words “Geo” meaning “earth” and “metry” meaning “measurement”. Is the given shape an example of a simple closed curve that is also a polygon? Solution: A closed shape that does not cross itself is a simple closed curve. Polygons are closed shapes formed of only straight lines like triangles, rectangles, pentagons and so on. The given figure is curved and is not made of only straight lines, this is not a polygon. In a triangle ABC, right angled at B, if ∠C=45° what is the measure of ∠A? Solution: △ ABC is the given right triangle with ∠B=90°. Sum of the angles of a triangle = 180° ∠A+∠B+∠C=180° ∠A+∠C=180°-90°=90° But, ∠C=45° hence ∠A=90°-45°=45° Identify the flat surfaces in the given prism. Solution: The flat surfaces in the prism are as given below: Rectangle AECB, Rectangle DCEF, Rectangle ABDF form the rectangular faces of the prism. △ BCD and △ AEF form the triangular faces of the prism. Attend this Quiz & Test your knowledge.ABCDCorrect answer is: BA closed shape that does not cross itself is a simple closed curve. It has the same start and end points.ABCDCorrect answer is: DThe given shape is a 3d shape and is called a cube.Correct answer is: 10 sides and 10 cornersThe star shaped figure is made of 10 straight lines. Thus, it has 10 sides and 10 corners. What is the relationship between mathematics and geometry? Mathematics is an umbrella term for various disciplines that focus on logic and abstract concepts. Geometry is one of the branches of mathematics that deals with shapes and sizes of figures and their properties. What are some applications of geometry? Geometry can be applied to many areas such as architecture, electronics, engineering, and construction. It can also be used in fields like science to develop projects or programs for space exploration. What is the main difference between 2D and 3D shapes? The main difference between 2D and 3D shapes is the absence of depth or height in 2D shapes. They are flat figures. 3D shapes exist in three dimensions with length, breadth, and height and do not appear flat. What is the difference between algebra and geometry? Algebra is a branch of mathematics that uses variables, in the form of letters, to act as numbers or quantities in equations and formulas. Geometry is a branch of mathematics that studies the sizes, shapes, positions, angles, and dimensions of things. Geometry is all about shapes and their properties. If you like playing with objects, or like drawing, then geometry is for you! Geometry can be divided into: Plane Geometry is about flat shapes like lines, circles and triangles ... shapes that can be drawn on a piece of paper Solid Geometry is about three dimensional objects like cubes, prisms, cylinders and spheres. Hint: Try drawing some of the shapes and angles as you learn ... it helps. A Point has no dimensions, only position A Line is one-dimensional A Plane is two dimensional (2D) A Solid is three-dimensional (3D) Why do we do Geometry? To discover patterns, find areas, volumes, lengths and angles, and better understand the world around us. Plane Geometry Plane Geometry is all about shapes on a flat surface (like on an endless piece of paper). General Drawing Tool Square Calculator and Rectangle Calculator Polygons A Polygon is a 2-dimensional shape made of straight lines. Triangles and Rectangles are polygons. Here are some more: Pentagon Pentagram Hexagon Properties of Regular Polygons Diagonals of Polygons Interactive Polygons The Circle Circle Theorems (Advanced Topic) There are many special symbols used in Geometry. Here is a short reference for you: Geometric Symbols Congruent Shapes Similar Shapes Types of Angles Using Drafting Tools Transformations: Rotation Reflection Translation Resizing Symmetry: Tessellations Tessellation Artist Pythagoras' Theorem Pythagorean Triples Circle Theorems Triangle Centers Trigonometry Trigonometry is a special subject of its own, so you might like to visit: Introduction to Trigonometry Trigonometry Index Solid Geometry is the geometry of three-dimensional space - the kind of space we live in ... ... let us start with some of the simplest shapes: Common 3D Shapes Polyhedra and Non-Polyhedra There are two main types of solids, "Polyhedra", and "Non-Polyhedra": Polyhedra (they must have flat faces): Cubes and Cuboids (Volume of a Cuboid) Platonic Solids Prisms Pyramids Investigating Solids Activity Non-Polyhedra (when any surface is not flat): Sphere Torus Cylinder Cone Copyright © 2025 Rod Pierce Geometry (from the Ancient Greek: geo- "earth", -metron "measurement") is a branch of mathematics, that is primarily concerned with the shapes and sizes of the objects, their relative position, and the properties of space. There are many postulates and theorems applied by the Greek mathematician Euclid, who is often referred to as the “Father of Geometry”. Let us explore all the important topics in Geometry. What is Geometry? Geometry is the branch of mathematics that relates the principles covering distances, angles, patterns, areas, and volumes. All the visually and spatially related concepts are categorized under geometry. There are three types of geometry: Euclidean Hyperbolic Elliptical Euclidean Geometry We study Euclidean geometry to understand the fundamentals of geometry. Euclidean Geometry refers to the study of plane and solid figures on the basis of axioms (a statement or proposition) and theorems. The fundamental concepts of Euclidean geometry include Points and Lines, Euclid’s Axioms and Postulates, Geometrical Proof, and Euclid’s Fifth Postulate. There are 5 basic postulates of Euclidean Geometry that define geometrical figures. A straight line segment is drawn from any given point to any other. A straight line is extended indefinitely in both directions. A circle is drawn with any given point as its center and any length as its radius. All right angles are congruent. Any two straight lines are infinitely parallel that are equidistant from one another at two points. Euclid's Axioms: Axioms or postulates are based on assumptions and have no proof for them. A few of Euclid's axioms in geometry that are universally accepted are: The things that are equal to the same things are equal to one another. If A = C and B = C then A = C If equals are added to equals, the wholes are equal. If A = B and C = D, then A + C = B + D If equals are subtracted, the remainders are equal. The coinciding things are equal to one another. The whole is greater than its part. If A > B, then there exists C such that A = B + C. The things that are double the same are equal to one another. The things that are halves of the same things are equal to one another. Non-Euclidean Geometry Spherical geometry and hyperbolic geometry are the two non-Euclidean geometries. Non-Euclidean geometry differs in its postulates on the nature of the parallel lines and the angles in the planar space, as validated by Euclidean geometry. Spherical geometry is the study of plane geometry on a sphere. Lines are defined as the shortest distance between the two points that lie along with them. This line on a sphere is an arc and is called the great circle. The sum of the angles in the triangle is greater than 180º. Hyperbolic geometry refers to a curved surface. This geometry finds its application in topology. Depending on the inner curvature of the curved surface, the planar triangle has the sum of the angles lesser than 180º. Plane Geometry Euclidean geometry involves the study of geometry in a plane. A two-dimensional surface extending infinitely in both directions forms the plane. Planes are used in every area of geometry and graph theory. The basic components of planes in geometry are analogous to points, lines, and angles. A point is the no-dimensional basic unit of geometry. Points lying on the same line are the collinear points. A line is a uni-dimensional unit that refers to a set of points that extends in two opposite directions and the line is said to be the intersection of two planes. A line has no endpoints. It is easy to differentiate a line, line segment, and ray. Lines may be parallel or perpendicular. Lines may or not intersect. Angles in Geometry When two straight lines or rays intersect at a point, they form an angle. Angles are usually measured in degrees. The angles can be an acute, obtuse, right angle, straight angle, or obtuse angle. The pairs of angles can be supplementary or complementary. The construction of angles and lines is an intricate component of geometry. The study of angles in a unit circle and that of a triangle forms the stepping stone of trigonometry. Transversals and related angles establish the interesting properties of parallel lines and their theorems. Plane Shapes in Geometry The properties of plane shapes help us identify and classify them. The plane geometric shapes are two-dimensional shapes or flat shapes. Polygons are closed curves that are made up of more than two lines. A triangle is a closed figure with three sides and three vertices. There are many theorems based on the triangles that help us understand the properties of triangles. In geometry, the most significant theorems based on triangles include Heron's formula, The exterior angle theorem, the angle sum property, the basic proportionality theorem, the similarity and Congruence in Triangles, the Pythagoras Theorem, and so on. These help us recognize the angle-side relationships in triangles. Quadrilaterals are polygons with four sides and four vertices. A circle is a closed figure and has no edges or corners. It is defined as the set of all points in a plane that are equidistant from a given point called the center of the circle. Various concepts centered around symmetry, transformations in shapes, construction of shapes are the formative chapters in geometry. Solid Geometry Solid shapes in geometry are three-dimensional in nature. The three dimensions that are taken into consideration are length, width, and height. There are different types of solid figures like a cylinder, cube, sphere, cone, cuboids, prism, pyramids, and so on and these figures acquire some space. They are characterized by vertices, faces, and edges. The five platonic solids and the polyhedrons have interesting properties in Euclidean space. The nets of the plane shapes can be folded into solids. Measurement in Geometry Measurement in geometry ascertains the calculation of length or distance, the area occupied by a flat shape, and the volume occupied by the solid objects. Mensuration in geometry is applied to the computation of perimeter, area, capacity, surface areas, and volumes of geometric figures. Perimeter is the distance around the plane shapes, the area is the region occupied by the shape, volume is the amount of region occupied by a solid, and the surface area of a solid is the sum of the areas of its faces. Two-dimensional Analytical Geometry Analytical geometry is popularly known as coordinate geometry is a branch of geometry where the position of any given point on a plane is defined with the help of an ordered pair of numbers, or coordinates using the rectangular Cartesian coordinate system. The coordinate axes divide the plane into four quadrants. Identifying and plotting points will be a building block of visualizing the geometric objects on the coordinate plane. In the example below, point A is defined as (4,3) and Point B is defined as (-3,1). The various properties of the geometric figures like straight lines, curves, parabolas, ellipse, hyperbola, circles, and so on can be studied using coordinate geometry. In analytical geometry, the curves are represented as algebraic equations, and this gives a deeper understanding of algebraic equations through visual representations. The distance formula, the section formula, midpoint formula, the centroid of a triangle, the area of the triangle formed by three given points, and the area of the quadrilateral formed by four points are determined using the known coordinates in the cartesian coordinate system. The equation of a straight line passing through a point, or two points, the angle between two straight lines are computed easily using the analytical geometry as they are generalized using formulas. Three-dimensional Geometry The three-dimensional geometry discusses the geometry of shapes in 3D space in the cartesian planes. Every point in the space is denoted by 3 coordinates, represented as an ordered triple (x, y,z) of real numbers. Direction Cosines of a Line If a straight line makes angles α, β and γ with the x-axis, y-axis, and z-axis respectively then cosα, cosβ, cosγ are called the direction cosines of a line. These are denoted as l = cosα, m = cosβ, and n = cosγ. For l, m, and n, l2 + m2 + n2 = 1, direction cosines of a line joining the points P(\(x_1, y_1, z_1\)) and Q(\(x_2, y_2, z_2\)) are given as :\(\dfrac{x_2-x_1}{PQ}, \dfrac{y_2-y_1}{PQ}, \dfrac{z_2-z_1}{PQ}\), where PQ = \(\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}\) Direction Ratios of a Line The directional ratios of a line are the numbers that are proportional to the direct cosines of the line. If l, m, n are the direction cosines, and a,b c are the direction ratios, then l = \(\dfrac{a}{\sqrt{a^2+b^2+c^2}}\), m = \(\dfrac{b}{\sqrt{a^2+b^2+c^2}}\) and n = \(\dfrac{c}{\sqrt{a^2+b^2+c^2}}\). Direction ratios of line joining the points P(\(x_1, y_1, z_1\)) and Q(\(x_2, y_2, z_2\)) are: \((x_2-x_1),(y_2-y_1), (z_2 -z_1)\) or \((x_1-x_2),(y_1-y_2), (z_1 -z_2)\) Skew lines in Geometry The skew lines are the lines in space that are neither parallel nor intersecting, and they lie in different planes. The angle between two lines is cos θ = |\(l_1l_2 + m_1m_2 + n_1n_2\)| where θ is the acute angle between the lines. Also Cos θ = |\(\dfrac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_1^2+b_1^2+c_1^2}}\)| Equation of Line in 3-D Geometry Vector equation of the line passing through a point with the position vector \(\vec a\) and parallel to vector \(\vec b\) is \(\vec r = \vec a+ \lambda \vec b\) Cartesian equation of the line passing through the point (\(x_1, y_1, z_1\)) and direction cosines l, m, n is \(\dfrac{x -x_1}{l} =\dfrac{y -y_1}{m}= \dfrac{z -z_1}{n}\) Vector equation of the line passing through two points with the position vectors \(\vec a\) and \(\vec b\) is \(\vec r = \vec a+ \lambda(\vec b -\vec a)\) Cartesian equation of the line passing through the points (\(x_1, y_1, z_1\)) and (\(x_2, y_2, z_2\)) is \(\dfrac{x -x_1}{x_2 -x_1} =\dfrac{y -y_1}{y_2 -y_1}= \dfrac{z -z_1}{z_2 -z_1}\) Angle Between Two Lines Angle between intersecting lines drawn parallel to each of the skew lines is the angle between skew lines. If θ is the angle between \(\vec r = \vec a_1+ \lambda \vec b_1\) and \(\vec r = \vec a_2+ \lambda \vec b_2\), then cos θ = |\(\dfrac{\vec b_1 . \vec b_2}{|\vec b_1| |\vec b_2|}\)| Also Check: Example 1. Given ABC and ADE are two triangles that are similar. Find the length of BC if AD = 7 units, DB = 3 units, AE = 4 units and DE = 7 units. Solution: In geometry, we know that similar triangles are proportional. In the given triangles, ABC and ADE are similar. Thus, AB/ AD = AC / AE = BC/ DE To find: BC Given: AB , AD, DB, DE and AE. AB = AD + DB = 7 + 3 = 10 units We know that AB/ AD = BC/ DE 10/7 = BC/7 70/7 = BC BC = 10 units Answer: The missing side BC = 10 units Example 2. What is the vertex of the given parabola? Solution: According to coordinate geometry, the vertex is the point where the axis of symmetry intersects the parabola. Here the parabola opens up, so the axis of symmetry is vertical. Looking at the graph we know that the axis of symmetry is the y-axis, which is x = -1. The y-coordinate is 3. Thus the vertex is at (-1, 3) Answer: The vertex of the given parabola is at (-1, 3) Example: 3 Find the direction cosines of the z-axis. Solution: In 3-dimensional geometry, if a straight line makes angles α, β and γ with the x-axis, y-axis, and z-axis respectively then cosα, cosβ, cosγ are called the direction cosines of a line. The z-axis makes 90º, 90º, and 0º with x, y, and z axes respectively. Thus their cosines are cos 90º, cos 90º, and cos 0º cos 90º = 0, cos 0º = 1. Therefore the direction cosines are 0,0,1 Answer: The direction cosines of the z-axis are 0,0,1. View More > go to slidego to slidego to slide Great learning in high school using simple cues Indulging in rote learning, you are likely to forget concepts. With Cuemath, you will learn visually and be surprised by the outcomes. Book a Free Trial Class FAQs on Geometry Geometry is the branch of mathematics that studies the shape, size, patterns, angle positions, dimensions, and properties of the objects around us and the spatial relationships among the objects. What are The Three Types of Geometry? The three types of geometry are Euclidean, Hyperbolic, and Elliptical Geometry. Who is The Father of Geometry? The father of Geometry is Euclid, a Greek mathematician. His book 'Elements' is the most influential work in maths serves as the fundamentals of geometry. It is a collection of propositions and postulates. The book consists of 13 volumes and it was second to the Bible in publications. What are The Five Basic Postulates of Euclidean Geometry? The five basic postulates are meant to be the fundamentals of geometry learned in the formative years. A straight line segment may be drawn from any given point to any other. A straight line may be extended indefinitely in both directions. A circle may be drawn with any given point as its center and any length as its radius. All right angles are congruent. Any two straight lines are infinitely parallel that are equidistant from one another at two points. What is LW in Geometry? LW is the formula used in geometry to find the area of a rectangle. Length and width are the parameters of a rectangle. The product of length and width is the area of the rectangle. Area of a rectangle = LW square units. How Do You Use Euclidean Geometry? Euclidean geometry studies the basic and complex geometric structures that are both plane shapes and solid shapes. Thus Euclidean geometry is used in art and architecture, computer science, astronomy, and other fields of mathematics. Euclidean geometry considers the study of points, lines, angles, and similarity and congruence in shapes, their patterns, and their transformations. Thus it has its practical applications in our day-to-day life. Why is it Called Hyperbolic Geometry? The 5th Euclidean postulate on parallel lines is not validated by the hyperbolic geometry. Hyperbolic geometry illustrates three key points that differ from Euclidean geometry. Two parallel lines converge in one direction and diverge in the other. The sum of angles in a triangle is less than 180º. Similar polygons of different areas don't exist. What Is The Difference Between Euclidean and Non-Euclidean Geometry? Euclidean Geometry is the study of the geometry of flat shapes on a plane, while non-Euclidean geometry aims at studying curved surfaces. What are the 2 Types of Geometry? Two types of geometry are plane geometry and solid geometry. Plane geometry deals with two-dimensional shapes and planes (x-axis and y-axis), while solid geometry deals with three-dimensional objects and 3D planes. These are the two types of geometry. What Is The Difference Between Solid and Plane Geometry? The study of shapes and objects that have length, width, and height is called the solid geometry, whereas the study of the shapes that have no thickness and can be represented only on a two-dimensional plane surface is called plane geometry. What is Elliptic Geometry Used For? Elliptic geometry studies the geometric structures that have curved surfaces. We use elliptic geometry to find the distance between the heavenly bodies in space, to calculate the distance between the places on the earth. How do you Teach Geometry? Teaching geometry is not an easy task that can be done with a textbook and drawing some shapes on the board. It requires a range of activities that demand learners' involvement to understand this concept with much more clarity. Some of the teaching-learning activities are listed below: Visualization- It involves learning through real-life experiences. We can take learners outside the classroom and help them to observe different shapes of objects, x-axis, and y-axis on the floor or any other flat surface, etc. Demonstration- It includes bringing some real-life objects to represent a concept in geometry. For example, it is always better to bring dice or any other object that represents a cubical shape to make learners understand the properties of a cube. DIY activities- We can ask learners to participate in some Do-It-Yourself activities that help them to work and play around with shapes and other concepts in geometry. Introducing Scientific Concepts- After all these activities, we can introduce the names and properties used in geometry using scientific terms. It includes introducing the terms like a cartesian plane, polyhedrons, quadrilaterals, etc. What is Geometry Divided Into? Geometry is the branch in mathematics that is further divided into various sub-branches that are given in the list below: Euclid’s Geometry Lines Angles Plane Shapes Solid Shapes Coordinate Geometry Vectors What are the Basics of Geometry? The basics of geometry are the proper understanding of points, lines, and planes. It then helps in building all other concepts in geometry that are based on these basic concepts. Q1: If an exterior angle of a triangle is 140 degrees and its opposite interior angles are equal to each other, which of the following is the measure of the equal angles of the triangle?30 degrees80 degrees50 degrees70 degreesQ2: In a right-angled triangle, the measure of an angle is 40°. Find the measure of other angles of the triangle in degrees.$$40° and 60°$$$$60° and 30°$$$$90° and 70°$$$$90° and 50°$$Q3: What is the supplementary angle of 30 degrees?60 degrees90 degrees120 degrees150 degreesQ4: How many sides does a hexagon has?10746Q5: A triangle has the measure of its angles as 70 degrees, 60 degrees, and 70 degrees. What is the type of triangle?ScaleneEquilateralIsoscelesThe triangle does not exist Science Mathematics geometry, the branch of mathematics concerned with the shape of individual objects, spatial relationships among various objects, and the properties of surrounding space. It is one of the oldest branches of mathematics, having arisen in response to such practical problems as those found in surveying, and its name is derived from Greek words meaning “Earth measurement.” Eventually it was realized that geometry need not be limited to the study of flat surfaces (plane geometry) and rigid three-dimensional objects (solid geometry) but that even the most abstract thoughts and images might be represented and developed in geometric terms.This article begins with a brief guidepost to the major branches of geometry and then proceeds to an extensive historical treatment. For information on specific branches of geometry, see Euclidean geometry, analytic geometry, projective geometry, differential geometry, non-Euclidean geometries, and topology. In several ancient cultures there developed a form of geometry suited to the relationships between lengths, areas, and volumes of physical objects. This geometry was codified in Euclid’s Elements about 300 bce on the basis of 10 axioms, or postulates, from which several hundred theorems were proved by deductive logic. The Elements epitomized the axiomatic-deductive method for many centuries. Analytic geometry was initiated by the French mathematician René Descartes (1596–1650), who introduced rectangular coordinates to locate points and to enable lines and curves to be represented with algebraic equations. Algebraic geometry is a modern extension of the subject to multidimensional and non-Euclidean spaces. Projective geometry originated with the French mathematician Girard Desargues (1591–1661) to deal with those properties of geometric figures that are not altered by projecting their image, or “shadow,” onto another surface. The German mathematician Carl Friedrich Gauss (1777–1855), in connection with practical problems of surveying and geodesy, initiated the field of differential geometry. Using differential calculus, he characterized the intrinsic properties of curves and surfaces. For instance, he showed that the intrinsic curvature of a cylinder is the same as that of a plane, as can be seen by cutting a cylinder along its axis and flattening, but not the same as that of a sphere, which cannot be flattened without distortion. Beginning in the 19th century, various mathematicians substituted alternatives to Euclid’s parallel postulate, which, in its modern form, reads, “given a line and a point not on the line, it is possible to draw exactly one line through the given point parallel to the line.” They hoped to show that the alternatives were logically impossible. Instead, they discovered that consistent non-Euclidean geometries exist. Topology, the youngest and most sophisticated branch of geometry, focuses on the properties of geometric objects that remain unchanged upon continuous deformation—shrinking, stretching, and folding, but not tearing. The continuous development of topology dates from 1911, when the Dutch mathematician L.E.J. Brouwer (1881–1966) introduced methods generally applicable to the topic. The earliest known unambiguous examples of written records—dating from Egypt and Mesopotamia about 3100 bce—demonstrate that ancient peoples had already begun to devise mathematical rules and techniques useful for surveying land areas, constructing buildings, and measuring storage containers. Beginning about the 6th century bce, the Greeks gathered and extended this practical knowledge and from it generalized the abstract subject now known as geometry, from the combination of the Greek words geo (“Earth”) and metron (“measure”) for the measurement of the Earth. In addition to describing some of the achievements of the ancient Greeks, notably Euclid’s logical development of geometry in the Elements, this article examines some applications of geometry to astronomy, cartography, and painting from classical Greece through medieval Islam and Renaissance Europe. It concludes with a brief discussion of extensions to non-Euclidean and multidimensional geometries in the modern age. The origin of geometry lies in the concerns of everyday life. The traditional account, preserved in Herodotus’s History (5th century bce), credits the Egyptians with inventing surveying in order to reestablish property values after the annual flood of the Nile. Similarly, eagerness to know the volumes of solid figures derived from the need to evaluate tribute, store oil and grain, and build dams and pyramids. Even the three abstruse geometrical problems of ancient times—to double a cube, trisect an angle, and square a circle, all of which will be discussed later—probably arose from practical matters, from religious ritual, timekeeping, and construction, respectively, in pre-Greek societies of the Mediterranean. And the main subject of later Greek geometry, the theory of conic sections, owed its general importance, and perhaps also its origin, to its application to optics and astronomy. While many ancient individuals, known and unknown, contributed to the subject, none equaled the impact of Euclid and his Elements of geometry, a book now 2,300 years old and the object of as much painful and painstaking study as the Bible. Much less is known about Euclid, however, than about Moses. In fact, the only thing known with a fair degree of confidence is that Euclid taught at the Library of Alexandria during the reign of Ptolemy I (323–285/283 bce). Euclid wrote not only on geometry but also on astronomy and optics and perhaps also on mechanics and music. Only the Elements, which was extensively copied and translated, has survived intact. Euclid’s Elements was so complete and clearly written that it literally obliterated the work of his predecessors. What is known about Greek geometry before him comes primarily from bits quoted by Plato and Aristotle and by later mathematicians and commentators. Among other precious items they preserved are some results and the general approach of Pythagoras (c. 580–c. 500 bce) and his followers. The Pythagoreans convinced themselves that all things are, or owe their relationships to, numbers. The doctrine gave mathematics supreme importance in the investigation and understanding of the world. Plato developed a similar view, and philosophers influenced by Pythagoras or Plato often wrote ecstatically about geometry as the key to the interpretation of the universe. Thus ancient geometry gained an association with the sublime to complement its earthy origins and its reputation as the exemplar of precise reasoning. Ancient builders and surveyors needed to be able to construct right angles in the field on demand. The method employed by the Egyptians earned them the name “rope pullers” in Greece, apparently because they employed a rope for laying out their construction guidelines. One way that they could have employed a rope to construct right triangles was to mark a looped rope with knots so that, when held at the knots and pulled tight, the rope must form a right triangle. The simplest way to perform the trick is to take a rope that is 12 units long, make a knot 3 units from one end and another 5 units from the other end, and then knot the ends together