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Horizontal line test

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The license may not give you all of the permissions necessary for your intended use. For example, other rights such as publicity, privacy, or moral rights may limit how you use the material. The horizontal line test is a convenient method that can determine if the inverse of a function is also a function. It is possible that the inverse of a function is not a function because it doesn't pass the vertical line test. So here's the deal! If the horizontal line intersects the graph of a function in all places at exactly one point, then the given function has an inverse that is also a function. We say this function passes the horizontal line test. Here are some examples of functions that pass the horizontal line test: Horizontal Line Cutting or Hitting the Graph at Exactly One Point Graph of the line $y = -x + 2$. Graph of the square root function $y = \sqrt{x + 1}$ Graph of the rational function $y = \frac{1}{x + 1}$. On the other hand, if the horizontal line can intersect the graph of a function in some places at more than one point, then the function involved can't have an inverse that is also a function. We say this function fails the horizontal line test. Here are some examples of functions that fail the horizontal line test: Horizontal Line Cutting or Hitting the Graph at More Than One Point Graph of the parabola $y = x^2 - 2$. Graph of absolute value function $y = |x|$ Graph of semi-circle $y = \sqrt{7 - x^2}$ You might also like these tutorials: Tags: Intermediate Algebra, Lessons The Horizontal line test is a simple way to see if a function is one-to-one (that is, it has exactly y-value corresponding with every x-value, or dependent value for every independent value). If a function passes the Horizontal line test, it has an inverse function. This is because the inverse of a function swaps the domain and range of the original function (the domain of the original function becomes the range of the inverse, and vice versa). The horizontal line test is a simple, visual way to tell if your function has an inverse function. It's useful because it tells us whether a function is one-to-one or not. More on that in a moment.Another benefit is that you get to draw with a colored pencil - a rare treat in math!Horizontal Line on a Graph One-to-One Function Examples Using the Horizontal Line Test Examples Challenge Practice Questions Question 1 Question 2 Question 3 Question 4 Question 5 Question 6 Question 7 Question 8 Question 9 Question 10 To Sum Up (Pun Intended)Horizontal Line on a GraphWhat's in a name? A horizontal line goes left to right across a page, like the horizon does on a nice sunny beach.It's called a test because just like a math test, there are two outcomes! A function can either pass or fail, depending on its shape. Read on, and you'll only be passing your next math test!One-to-One FunctionExamples are math machines. Give them an input, and they give you an output.Think of functions as ovens: put in food, and they give you a cooked meal!If each specific input corresponds to a single specific output, the function is one-to-one. One input is converted to one output.Functions can also be many-to-one, where several inputs have the same single output. It's like your cake recipe: there are lots of ways to make the cake mix, but some of them give the same delicious result!You want to be certain what food your oven makes, so you want a clear output. It would be difficult to cook with an oven which gives different food from the same ingredients!You might come across one-to-many or many-to-many relations but, strictly speaking, these aren't functions. Remember that each input must have only one output to be an official function.Examples $y = x$ is a one-to-one function because the output is exactly the same as the input. There's no way for two numbers to be simultaneously the same and different! $y = \sin(x)$ is not one-to-one because it has a repeating pattern.For a clear example, $y = \sin(0^\circ) = 0$ but $y = \sin(360^\circ) = 0$ too, so it is still a function, but is one-to-many. $y = x^2$ is not one-to-one because each number's negative partner is sent to the same output. Take the inputs 1 and -1, then $y = 1^2 = 1$ and $y = (-1)^2 = 1$ both equal 1! $y = \sqrt{x}$ is one-to-one because the square root of a negative number is not real.The only real outputs are positive, with each input value neatly giving one output.Whether the function is one-to-one, or even a function can be difficult to see just by looking at the equation. That's where the horizontal line test comes in.Using the Horizontal Line TestThe test is actually more of a rule! It goes like this:If all horizontal lines intersect the function's graph at a single point, or no points, then the function passes the test.If you can find even one horizontal line which intersects the function's graph at more than one point, then the function fails the test. Get a suitably zoomed-out copy of your graph, showing all the important features of its shape.Hold a ruler horizontally - remember horizontal - across the graph and move it up and down. Are there any positions where the edge of the ruler intersects the graph at more than one point?If there aren't any horizontal lines that would intersect the graph in more than one place, you're done. Draw some evenly spaced horizontal lines to demonstrate your result.If you've found a position where a horizontal line intersects the graph at more than one point, draw it on!The important features to include in your graph are:All x- and y-intercepts - where the graph crosses the axes.Maximum and minimum points - where the graph changes direction from increasing to decreasing, or the other way around behavior and asymptotes - where the graph will continue to and where it never reaches. If there are no more changes of direction, then you can stop there.Any repeatsThe test also works on computers! Either hold a straight edge up to the screen or imagine a sliding horizontal line.Some resources, like Desmos, will let you slide a horizontal line across graphs! It's a fantastic way to visualize it.Examples $y = x^3 + 1$ passes the test. The middle section is a close call, but is luckily never completely flat because $f(x)$ is always increasing as x increases.There's no "dipping up" or "dipping down" in the graph which would intersect with a horizontal line more than once.By drawing a few "test" horizontal lines in your favorite, colored pen, you can clearly see that none of these could ever intersect the graph at more than one point! $y = \frac{1}{x}$ passes the test. The part on the left, where $x < 0$, is entirely beneath the y-axis. The other part, where $x > 0$, is entirely above the y-axis.These parts cannot ever have the same y-value!Just like before, drawing some spaced-out horizontal lines shows that both parts of the graph pass the test. $y = \cos(x)$ fails the horizontal line test spectacularly. Drawing a horizontal line anywhere within $(-1, 1)$ gives an infinite number of intersections with the function's graph!Remember that to pass the test, every possible horizontal line must intersect the graph at one or zero points.ChallengeThe function $y = x$ passes the horizontal line test.What happens if you shift the graph up or down by adding a constant? Challenge Answer (Open to See) $y = x + c$ always passes the test, for any constant c !You can test this for yourself by trying several shifts like $y = x + 1$ and $y = x - 10$.The shape of the graph doesn't change, so there are no "dips" where a horizontal line could intersect with it at more than one point.When the constant term is added, the horizontal lines can simply be moved up or down on your page. You'd end up with a graph that looks almost the same!Practice QuestionsThe Matter of Math mantra is practice, practice, practice! Determine for yourself whether the following functions are one-to-one or not, using the horizontal line test.The Matter of Math mantra is practice, practice, practice!Don't forget, a function is only one-to-one if it passes the test, and cannot be one-to-one if it fails!The questions start gentle but will get more difficult, so be careful when plotting the graphs.Question 1s $y = 3x + 5$ one-to-one? This function gives a straight-line graph. This passes the test, so it is one-to-one.Question 2s $y = x + 9$ one-to-one? This graph is still a straight line but is "facing the other way". It also passes the test and is one-to-one.Question 3s $y = 3x^2 + 4x$ one-to-one? This graph looks very similar to the $y = x^2$ example you saw at the start of the lesson. It fails the test so cannot be one-to-one.Question 4s $y = \tan(x)$ one-to-one? Similar to $\cos(x)$ and $\sin(x)$, $\tan(x)$ has a pattern that repeats every π , so it fails the test and is not one-to-one.But it does look pretty cool doing it! Any horizontal line intersects with the graph at an infinite number of points.Question 5s $x^2 + y^2 = 4$ one-to-one?Do you recognize this equation? Hint: find out what shape it gives to help you draw a graph. It is a circle! Centered at the origin with a radius of 2. Any horizontal line passing through the inside of the circle must intersect it at two points, so this function is not one-to-one and fails the test!Question 6s $y = (x - 2)^3$ one-to-one? This function passes the test and is one-to-one because it's the same as $y = x^3$ but translated to the right. If you'd like to see why translation does not affect the result, check out the challenge!Question 7s $y = \frac{1}{9x}$ one-to-one? It is one-to-one because it passes the test. This function is a transformation of $y = \frac{1}{x}$, which you already know passes the test.It's a little steeper than its $1/x$ cousin, but the transformation doesn't add any strange twists or dips to its shape!Question 8s $y = \sin(x)$ one-to-one?Past... confused by the notation here? Math Shorts has a little crash course for you! The graph fails the test so this cannot be a one-to-one function. Just like $y = x^2$, you've got a reflection in the y-axis.Question 9s $y = \sin(x + \pi)$ one-to-one? π looks a bit scary, but it's just a number, so it shifts the whole graph to the left by π , or 3.14159... This doesn't affect the shape so just like the regular $y = \sin(x)$ function, $y = \sin(x + \pi)$ fails the test and is not a one-to-one function.Question 10s $y = \sqrt{6 - x}$ one-to-one? A very similar example to earlier! Yet again, you've got a transformation of a familiar function. As you move from left to right, the function is always decreasing. This means the graph doesn't have any waves so it passes the horizontal line test and is one-to-one.To Sum Up (Pun Intended)!That's it for the horizontal line test! Another handy test you're likely to meet soon if you haven't already is the vertical line test.Functions are like "math ovens", producing outputs depending on what you put into them!A graph passes the horizontal test if every horizontal line crosses, or intersects it, at one or fewer points. If the graph passes the test, it is a one-to-one function - every specific input corresponds to exactly one output.Functions can also be many-to-one, but many-to-many and one-to-many relations are technically not actually functions.I hope you enjoyed this lesson. Please leave a comment at the bottom to let us know how you found the lesson, or for some help with any bits you're not sure on, help is at hand! Learning Outcomes Use the vertical line test to determine if a graph represents a function Use the horizontal line test to determine if a graphed function is one-to-one As we saw in the previous section, functions can be represented by graphs. Graphs are useful as they display many input-output pairs in a small space and the visual information they provide can make relationships clearer. We typically construct graphs with the domain values along the horizontal axis and the range values along the vertical axis. This is because the domain values (e.g. x) are independent and independent variables are generally associated with the horizontal axis. The values in the range are called function values and are dependent on the domain values and are therefore associated with the vertical axis. Since the rectangular coordinate system is most commonly associated with an x -axis and a y -axis, along with coordinate pairs (x, y) , it is common when first learning functions to name the domain values x and the function values y , and we say y is a function of x , or $y = f(x)$ when the function is named f . The graph of the function is the set of all points (x, y) in the plane that satisfies the equation $y = f(x)$. If the function is defined for only a few input values, then the graph of the function is only a few points, where the x -coordinate of each point is a domain value and the y -coordinate of each point is the corresponding function value. In other words, if $y = f(x)$, then the coordinates of any point on the graph of $y = f(x)$ are of the form $(x, f(x))$. For example, the coordinates of the black dots on the graph in figure 1 are $(0, 2)$ and $(6, 1)$. This tells us that the function value when $x = 0$ is 2, and the function value when $x = 6$ is 1. In function notation, this is written $f(0) = 2$ and $f(6) = 1$. The points $(0, 2)$ and $(6, 1)$ are only two points on the graph. The graph of the set of all points $(x, f(x))$ is a curve. The curve shown includes $(0, 2)$ and $(6, 1)$ because the curve passes through those points. Figure 1. Graph of a function Some Graphs Are Not Functions Any function $y = f(x)$ can be represented by a graph. However, every graph does not represent a function. Remember that a function is a relation with a one-to-one or many-to-one mapping. All graphs represent relations, but that relation may not be a function. Consider the graph in figure 1. Choose any x -value and it is clear that it has a single corresponding y -value. There is no x -value that maps to two or more y -values. This is the definition of a function. Now consider the graph in figure 2. Figure 2. Graph does not represent a function The only x -value that has a single y -value is $x = -4$. Any other x -value we choose has 2 y -values. This means that the mapping of x to y is one-to-many and therefore this graph does not represent a function. The Vertical Line Test The vertical line test can be used to determine whether a graph represents a function. A vertical line includes all points with a particular x -value. If we can draw any vertical line that intersects a graph more than once, then the graph does not define a function because that x -value has more than one y -value. For each x -value in the domain, a function has only one y -value (or function value) in the range. The vertical line test can be used to determine if a graph represents a function. Inspect the graph to see if any vertical line drawn would intersect the curve more than once. If any vertical line intersects the graph more than once, the graph does NOT represent a function. If any vertical line intersects the graph only once, the graph does represent a function. Which of the graphs represents(s) a function $y = f(x)$? Solution If any vertical line intersects a graph more than once, the relation represented by the graph is not a function. Notice that any vertical line would pass through only one point of the two graphs shown in parts (a) and (b) above. From this we can conclude that these two graphs represent functions. The third graph, (c), does not represent a function because a vertical line intersects the graph at more than one point. Does the graph represent a function? In the graph below, use the vertical line test to determine which relations are functions and which are not. In the previous section, we learned that a function is a relation with a one-to-one or many-to-one mapping. The vertical line test can tell us if a graph represents a function, but how will we know if that function is one-to-one? In order for a function to be one-to-one, not only does every x -value map to only one y -value but every x -value is mapped from exactly one x -value. Consider the graph in figure 3. We can see it is a function as any vertical line passes through only one point. But is the function one-to-one? If we choose any x -value, say $x = 4$, we can see that the points $(-2, 4)$ and $(2, 4)$ both lie on the curve. This means that this function is not one-to-one as the x -value 4 is mapped from two different x -values, -2 and 2. Figure 3. A function that is not one-to-one One we have determined that a graph defines a function, an easy way to determine if it is a one-to-one function is to use the horizontal line test. A horizontal line includes all points with a particular y -value. If we can draw any horizontal line that intersects the graph more than once, then the graph does not represent a one-to-one function because that y -value is mapped from more than one x -value. How To: Given a graph of a function, use the horizontal line test to determine if the graph represents a one-to-one function. Inspect the graph to see if any horizontal line drawn would intersect the curve more than once. If there is any such line, the function is not one-to-one. If no horizontal line can intersect the curve more than once, the function is one-to-one. Determine if the graph represents a one-to-one function. Solution (a) The graph represents a function since any vertical line passes through only one point on the graph. But the function is not one-to-one. The horizontal line shown below intersects the graph of the function at two points (and we can even find horizontal lines that intersect it at three points.) (b) The graph represents a one-to-one function as any vertical and horizontal lines pass through only one point on the graph. (c) The graph does not represent a function as a vertical line can pass through more than one point. Since it is not a function, it cannot be a one-to-one function.; Determine if the graph represents a one-to-one function. 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